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Math54 Final Exam, Fall 2016

This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total	100	

Your Name: Richard Sim

Your GSI: Eric Hallman

Your SID: 26077217

Richardson

1)
$$\left[\begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ -3 & 1 & 9 & -5 & 9 \\ 4 & -8 & -1 & 7 & 0 \end{array}\right] \sim \begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ 0 & 16 & 3 & -5 & -12 \\ 0 & -28 & 7 & 7 & 28 \end{array} \sim \begin{array}{ccccc} 1 & 5 & -2 & 0 & -7 \\ 0 & 16 & 3 & -5 & -12 \\ 0 & 1 & -1/4 & -1/4 & -1 \end{array}$$

$$\sim \begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ 0 & 1 & 3/16 & -\frac{5}{16} & -\frac{12}{16} \\ 0 & 1 & -1/4 & -1/4 & -1 \end{array} \sim \begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ 0 & 1 & 3/16 & -\frac{5}{16} & -\frac{3}{4} \\ 0 & 0 & -1/4 - 3/16 & -\frac{1}{4}+\frac{5}{16} & -1+\frac{3}{4} \\ & & -\frac{4}{16}-\frac{3}{16} & -\frac{4}{16}+\frac{5}{16} & -\frac{1}{4} \end{array}$$

$$\sim \begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ 0 & 1 & \frac{3}{16} & -\frac{5}{16} & -\frac{3}{4} \\ 0 & 0 & -\frac{7}{16} & \frac{1}{16} & -\frac{1}{4} \end{array} \sim \begin{array}{cccc|c} 1 & 5 & -2 & 0 & -7 \\ 0 & 1 & 0 & -\frac{2}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & -\frac{1}{7} & \frac{4}{7} \end{array} \sim \begin{array}{cccc|c} 1 & 5 & 0 & -\frac{2}{7} & -7+\frac{8}{7} \\ 0 & 1 & 0 & -\frac{2}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & -1/7 & 4/7 \end{array}$$

$$\frac{3}{16} + -\frac{7}{16}\left(\frac{3}{7}\right) = 0$$

$$-\frac{5\cdot 7}{16\cdot 7} + \frac{1}{16}\left(\frac{3}{7}\right) = \frac{-\cancel{32}\,2}{\cancel{16}\cdot 7} = -\frac{2}{7}$$

$$-\frac{3\cdot 7}{4\cdot 7} + \left(-\frac{1}{4}\right)\left(\frac{3}{7}\right) = -\frac{21}{28} - \frac{3}{28} = -\frac{\cancel{24}\,6}{\frac{28}{14}\,7} = -\frac{6}{7}$$

$$\sim \begin{array}{cccc|c} 1 & 0 & 0 & -3+\frac{10}{7} & -\frac{49}{7}+\frac{8}{7}+\frac{30}{7} \\ 0 & 1 & 0 & -2/7 & -\frac{6}{7} \\ 0 & 0 & 1 & -1/7 & 4/7 \end{array}$$

$-57+30$

$$\sim \begin{array}{cccc|c} 1 & 0 & 0 & +\frac{8}{7} & -\frac{11}{7} \\ 0 & 1 & 0 & -\frac{2}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & -1/7 & 4/7 \end{array}$$

$$\boxed{x = \begin{bmatrix} -\frac{11}{7} \\ -\frac{6}{7} \\ \frac{4}{7} \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -\frac{8}{7} \\ \frac{2}{7} \\ \frac{1}{7} \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} -8 \\ 2 \\ 1 \\ 7 \end{bmatrix}$$

$-8+10-2=0$ ✓

$24+2+9-35=0$ ✓

$-32-16-1+44=0$ ✓

2. Let matrices $A, L, U \in \mathcal{R}^{n\times n}$ satisfy

$$A = L\,U,$$

where L is invertible lower triangular and U is upper triangular.

(a) Show that the first column of A is a multiple of the first column of L.

(b) For any $1 \le k \le n$, show that the k-th column of A is a linear combination of the first k columns of L.

$A = LU$

$$A = \begin{bmatrix} * & & 0 \\ \vdots & \ddots & \\ * & \cdots & * \end{bmatrix} \begin{bmatrix} * & \cdots & * \\ & \ddots & \vdots \\ 0 & & * \end{bmatrix}$$

know

$$A = [a_1, a_2 \ldots a_n] = [Lu_1, Lu_2 \ldots Lu_n]$$

a_1 is first column of A

$Lu_1 = a_1$

Since u_1 is upper triangular, and is the first vector, in order to be upper triangular, u_1 must have only 1 entry that is first on the column. ∴ the rest of the vector is 0

$$Lu_1 = l_1 u_{1,1} + l_2(0) \ldots l_n(0)$$
$$= l_1 u_{1,1} + 0 + \ldots + 0$$
$$= l_1 u_{1,1} = a_1$$

∴ $a_1 = l_1 u_{1,1}$ where $u_{1,1}$ is any multiple and we can see that a_1 is a scalar multiple of the first column of L.

$$A = [a_1, a_2 \ldots a_n] = [Lu_1, Lu_2 \ldots Lu_n]$$

where $a_k = L u_k \rightarrow$ u_k - column of U, a_k - column of A

for any vector of an upper triangular matrix, u_k must have at least $n-k$ elements from the bottom up as 0. The rest of the k elements can be 0 or nonzero from the top of the matrix $(u_{k,1} \ldots u_{k,k})$

from matrix vector mult,

$$Lu_k = l_1 u_{k,1} + \ldots + l_k u_{k,k} + \ldots l_n u_{k,n}$$

since rest of elements from $k \to n$ are 0 for $u_{k+1,k+1}$, $u_{n,n}$,

$$= l_1 u_{k,1} + \ldots + l_k u_{k,k} + 0 + \ldots + 0 = a_k$$

$a_k = l_1 u_{k,1} + \ldots + l_k u_{k,k}$

∴ we can see that a_k is a lin comb of $l_1 \ldots l_k$ (first k col of L) where the scalar mult are $u_{k,1} \ldots u_{k,k}$

3. Let V be the space $C[-1,1]$, define the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(x)\, g(x)dx,$$

for any $f, g \in C[-1,1]$. Find an orthogonal basis for the subspace spanned by the polynomials $1, x$ and x^2.

$1, x, x^2$

gram schmidt

$\vec{V}_1 = 1$

$\vec{V}_2 = x - \frac{\langle 1, x\rangle}{\langle 1,1\rangle} 1$

$\vec{V}_3 = x^2 - \frac{\langle x^2, \vec{V}_2\rangle}{\langle \vec{V}_2, \vec{V}_2\rangle} V_2 - \frac{\langle x^2, 1\rangle}{\langle 1,1\rangle} 1$

$\vec{V}_2 = x - \frac{\int_{-1}^{1} x\,dx}{\int_{-1}^{1} dx} = x - \frac{\frac{x^2}{2}\Big|_{-1}^{1}}{x\Big|_{-1}^{1}} = x - \frac{\left(\frac{1}{2} - \frac{1}{2}\right)}{1-(-1)=2} = x$

$\vec{V}_3 = x^2 - \frac{\langle x^2, x\rangle}{\langle x, x\rangle} x - \frac{\langle x^2, 1\rangle}{\langle 1, 1\rangle}$

$= x^2 - \frac{\int_{-1}^{1} x^3 dx}{\int_{-1}^{1} x^2 dx} x - \frac{\int_{-1}^{1} x^2 dx}{\int_{-1}^{1} dx}$

$= x^2 - \frac{\frac{x^4}{4}\Big|_{-1}^{1}}{\frac{x^3}{3}\Big|_{-1}^{1}} x - \frac{\frac{x^3}{3}\Big|_{-1}^{1}}{x\Big|_{-1}^{1}}$

$= x^2 - \frac{\cancel{\frac{1^4}{4} - \frac{(-1)^4}{4}}\ 0}{\cancel{\frac{1^3}{3} + \frac{(1)^3}{3}}} x - \frac{\frac{1^3}{3} - \frac{(-1)^3}{3}}{1+1}$

$= x^2 - \frac{\frac{2}{3}}{2}$

$= x^2 - 1/3$

orthog basis:

$$\boxed{\{1, x, x^2 - 1/3\}}$$

↳ spans $1, x, x^2$

↳ lin. indep b/c

$\left[\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ from looking at coord.

check:

$\int_{-1}^{1} 1(x)\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = \frac{1}{2} - \frac{(-1)^2}{2} = 0$ ✓

$\int_{-1}^{1} x^2 - 1/3\, dx = \frac{x^3}{3} - \frac{x}{3}\Big|_{-1}^{1} = \frac{x^3 - x}{3}\Big|_{-1}^{1}$

$= \frac{\cancel{1-1}}{3}\,0 - \frac{\cancel{((-1)+1)}}{3}\,0 = 0$ ✓

$\int_{-1}^{1} x(x^2 - 1/3)\,dx = \int_{-1}^{1} x^3 - \frac{x}{3}\,dx$

$= \frac{x^4}{4} - \frac{x^2}{6}\Big|_{-1}^{1} = \cancel{\tfrac{1}{4}} - \cancel{\tfrac{1}{6}} - \cancel{\tfrac{1}{4}} + \cancel{\tfrac{1}{6}} = 0$

✓

4. Let $A \in \mathcal{R}^{m\times n}$ and $B \in \mathcal{R}^{n\times p}$ be matrices such that

$$AB = 0.$$

(a) Show that the column space of B is a subspace of the null space of A.

(b) Show that $\mathbf{rank}(A) + \mathbf{rank}(B) \le n$.

From matrix product $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & & \\ a_{m1} & & a_{mn} \end{bmatrix}$

$[Ab_1, Ab_2, \ldots Ab_p] = 0$

↳ columns of B are in nullspace of A in order to get $\vec{0}$ as $n\times p$ matrix

∴ $Ab_1 = \vec{0}, \ldots Ab_p = \vec{0}$ or else it wont be $\vec{0}$ $n\times p$ matrix

$A(b_1 + \ldots + b_n)$

$= A(b_1) + \ldots + A(b_p) = \vec{0}$ b/c A is a matrix transformation, it is linear

closed to scaling

$A(c_1 b_1) + \ldots + A(c_p b_p) = 0$

$A(c_1 b_1 + \ldots + c_p b_p) = \vec{0}$

shows that column space of B is closed to addition b/c:

$A(b_1 + \ldots + b_p) = Ab_1 + \ldots + Ab_p = 0$ $b_1 + \ldots + b_p$ B in closed subspace,

$A(c_1 b_1 + \ldots + c_p b_p) = \underset{0}{A(c_1 b_1)} + \ldots + \underset{0}{A(c_p b_p)} = 0$] closed to scaling &

$c_1 b_1 + \ldots + c_p b_p$ is in nullspace of A

$\text{col } B = c_1 b_1 + \ldots + c_p b_p$ so columnspace of B is subspace of nullspace of A & it is closed to addition & scaling also has 0 vector when $c_1, \ldots c_p = 0$

∴ subspace.

Thoughts:

↳ $\text{rank}(A) + \text{rank}(B) \le n$

rank A is at most, m dimensional $\dim A \le m$ $\quad$ $\text{rank } A + \dim \text{nul} A = n$

Soln: rank B is at most, n dimensional $\dim B \le n$

$\text{rank } A + \text{nul } A = n$ rank nullity thm.

know: $\dim \text{col} B \le \dim \text{nul} A$ → $\dim \text{col } B \le \dim \text{nul } A$ b/c a subspace cannot be higher dim than the vectorspace it is a part of.

$\text{rank} B \le n - \text{rank} A$ from rank nullity thm.

∴ $\boxed{\text{rank} A + \text{rank} B \le n}$ due to limitation of subspace

5. Solve the given initial value problem

$$y'' + y' = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Char eq:

$r^2 + r = 0$

$r(r+1) = 0$

$r = 0, -1$

ansaatz $Y_h(t) = C_1 e^{0t} + C_2 e^{-t}$

$Y_h'(t) = -C_2 e^{-t}$

$Y_h'(0) = -C_2 e^{0}$

$= -C_2$

$= 0$

$-C_2 = 0$

$C_2 = 0$

$Y_h(t) = C_1 + C_2 e^{-t}$

$Y_h(0) = 1 = C_1 + C_2 e^{-0}$

$1 = C_1 + C_2$

$C_1 = 1$

$\boxed{Y_h(t) = 1}$

$y'(t) = 0$

$y''(t) = 0$

$0 + 0 = 0$ ✓

$y(0) = 1$ ✓

$y'(0) = 0$ ✓

~~L~~6. Let

0

$$A = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}.$$

(a) Find a general solution of

$$\mathbf{x}'(t) = A\mathbf{x}(t).$$

(b) Solve the initial value problem

$$\mathbf{x}'(t) = A\mathbf{x}(t), \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

a

$A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$

$A - I\lambda = \begin{matrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{matrix}$

$\det(A - I\lambda) = -(2-\lambda)(2+\lambda)+3$

$= -(4 + 2\lambda - 2\lambda - \lambda^2) + 3$

$= -4 + \lambda^2 + 3$

$= \lambda^2 - 1$

$\lambda^2 - 1 = 0$

$\sqrt{\lambda^2} = \sqrt{1}$

$\lambda = \pm 1$

$\lambda = 1$

$\begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} 3x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\lambda = -1$

$\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\vec{x}(t) = c_1 e^{t} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

check homogeneous only

$x'(t) = c_1 e^{t} \begin{bmatrix} 3 \\ 1 \end{bmatrix} - c_2 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3c_1 e^{t} - e^{-t} c_2 \\ c_1 e^{t} - c_2 e^{-t} \end{bmatrix}$

checked on another sheet ✓

b

$\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{B} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$B^{-1} = \frac{1}{\det B} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{3-1} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$c_1 = 0$

$c_2 = 1$

$\boxed{\vec{x}(t) = e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

check $\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

_hecking for 6

$$\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3e^t + e^{-t} \\ e^t + e^{-t} \end{bmatrix}$$

$3e^t - e^{-t}$

$$\begin{bmatrix} 2(3e^t + e^{-t}) - 3(e^t + e^{-t}) \\ 3e^t + e^{-t} - 2e^t - 2e^{-t} \end{bmatrix} = \begin{bmatrix} 6e^t + 2e^{-t} - 3e^t - 3e^{-t} \\ e^t - e^{-t} \end{bmatrix}$$

$$= \begin{matrix} 3e^t - e^{-t} \\ e^t - e^{-t} \end{matrix}$$

7. (a) Find the values of the positive parameter λ for which the given problem below has a nontrivial solution.

$$y'' + \lambda y = 0 \quad \text{for} \quad 0 < x < \pi; \quad y(0) = 0, \; y'(\pi) = 0.$$

(b) Compute the Fourier Cosine series of the function $f(x) = x$ on the interval $[0, \pi]$.

$y'' + \lambda y = 0$

$r^2 + \lambda = 0$

$\sqrt{r^2} = \sqrt{-\lambda}$

$r = \pm\sqrt{-\lambda}$

if $\lambda < 0$

$y(t) = C_1 e^{\sqrt{-\lambda} t} + C_2 e^{-\sqrt{-\lambda} t}$

$y(0) = C_1 + C_2 = 0$

$C_1 + C_2 = 0$

$C_1 = -C_2$

$y'(t) = C_1\sqrt{-\lambda} e^{\sqrt{-\lambda} t} - C_2\sqrt{-\lambda} e^{-\sqrt{-\lambda} t}$

$y'(\pi) = C_1\sqrt{-\lambda} e^{\sqrt{-\lambda}\pi} - C_2\sqrt{-\lambda} e^{-\sqrt{-\lambda}\pi}$

$y'(\pi) = -C_2\sqrt{-\lambda} e^{\sqrt{-\lambda}\pi} - C_2\sqrt{-\lambda} e^{-\sqrt{-\lambda}\pi}$

$= -C_2\underbrace{(\sqrt{-\lambda} e^{\sqrt{-\lambda}\pi} + \sqrt{-\lambda} e^{-\sqrt{-\lambda}\pi})}_{\text{never} \neq 0}$

$= 0$

$C_1 = 0$ $\quad C_2 = 0$ nope

if $\lambda = 0$ $\quad r = \pm 0$

$y(t) = C_1 e^0 + C_2 t e^{0t}$

$= C_1 + C_2 t$

$y'(t) = C_2$

$y(0) = C_1 = 0$

$y'(\pi) = C_2 = 0$ nope

$\frac{2\pi 2}{2n+1} = \frac{4\pi}{2n+1}$

$= 4\pi$ period

$\lambda > 0$

$r = \pm i\sqrt{\lambda}$

$y(t) = C_1\cos(\sqrt{\lambda} t) + C_2\sin(\sqrt{\lambda} t)$

$y'(t) = -C_1\sqrt{\lambda}\sin(\sqrt{\lambda} t) + C_2\sqrt{\lambda}\cos(\sqrt{\lambda} t)$

$y(0) = 0 = C_1 \overset{(1)}{\cos(0)} + C_2 \overset{0}{\cancel{\sin(0)}}$

$C_1 = 0$

$y'(t) = -C_1\sqrt{\lambda}\overset{0}{\cancel{\sin(\sqrt{\lambda} t)}} + C_2\sqrt{\lambda}\cos(\sqrt{\lambda} t)$

$= C_2\sqrt{\lambda}\cos(\sqrt{\lambda} t)$

$y'(\pi) = 0 = C_2\sqrt{\lambda}\cos(\sqrt{\lambda}\pi)$

$\cos(\sqrt{\lambda}\pi) = 0 \rightarrow$ so we can get nonzero C_2

$\sqrt{\lambda}\pi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\sqrt{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

$\sqrt{\lambda} = \frac{2n+1}{2}$

$n = 0, 1, 2, \ldots$

$\boxed{\lambda = \left(\frac{2n+1}{2}\right)^2}$

yea

assuming I'm writing for regular one w/ 2L periodicity

asked Eric about different periodicity $f(x)$

$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

where

$a_0 = \frac{2}{\pi}\int_0^\pi x\,dx = \frac{2}{\pi}\frac{x^2}{2}\Big|_0^\pi = \frac{\pi^2}{\pi} = \pi$

$\boxed{a_0 = \pi}$

$a_n = \frac{2}{\pi}\int_0^\pi x\cos\left(\frac{n\pi}{\pi}x\right)dx$

$= \frac{2}{\pi}\int_0^\pi x\cos(nx)\,dx$ $\quad u = x$, $du = dx$ $\quad dv = \cos(nx)\,dx$, $v = \frac{1}{n}\sin(nx)$

$= \frac{2}{\pi}\left(\frac{x}{n}\sin(nx) - \int_0^\pi \frac{1}{n}\sin(nx)\,dx\right)$

$= \frac{2}{\pi}\left(\frac{x}{n}\sin(nx) - \frac{1}{n^2}(-\cos(nx))\right)\Big|_0^\pi$

$= \frac{2}{\pi}\left(\frac{x}{n}\sin(nx) + \frac{\cos(nx)}{n^2}\right)\Big|_0^\pi$

$= \frac{2}{\pi}\left(\frac{\pi}{n}\overset{0}{\sin(n\pi)} + \frac{\cos(n\pi)}{n^2} - 0 - \frac{\cos(0)}{n^2}\right)$

$= \frac{2}{\pi}\left(\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2}\right)$

$= \frac{2}{\pi}\left(\frac{\cos(n\pi) - 1}{n^2}\right)$ $\quad n = 1, 2 \ldots$

$= \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$ $\quad n = 1, 2 \ldots$

$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx$

where

$a_0 = \pi$

$a_n = \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$