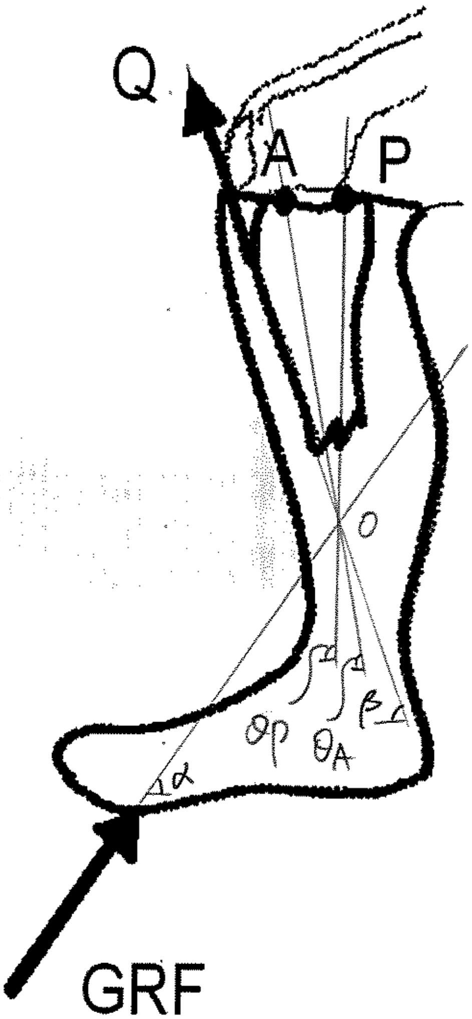
Orthopedic Biomechanics
Fall 2017 Midterm

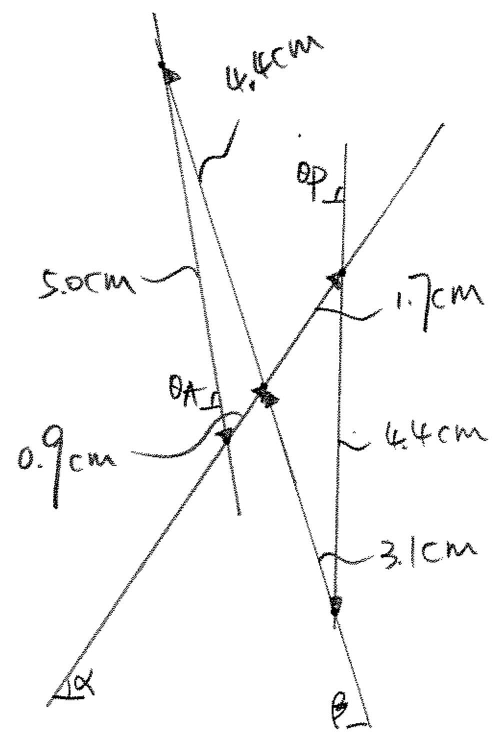
Name: Solution

1. [25 points total] Statics
Here, we analyze the effects of changing the location of the tibial joint contact force **J** from a posterior point **P** to an anterior point **A** within the knee joint as depicted in the picture below. For both cases, assume that the ground reaction force **GRF** is unchanged, and that the *orientation* of the quadriceps muscle force **Q** is also unchanged. Treat this as a 2D statics problem and ignore the weight of the leg.

i) [15 points] Use the picture below and a graphical *three-force vector triangle* approach to estimate the change in the magnitude of the muscle force **Q** when the contact point on the tibia changes from point **A** to point **P**. Express your answer as a ratio (value for point **A** / value for point **P**).



Three-force vector:



---- (10)

$$\frac{Q_A}{GRF} = \frac{5.0\,cm}{0.9\,cm}$$

$$\frac{Q_P}{GRF} = \frac{3.1\,cm}{1.7\,cm}$$

$$\therefore \frac{Q_A}{Q_P} = \frac{5.0/0.9}{3.1/1.7} = \boxed{3.0}$$ ---- (5)

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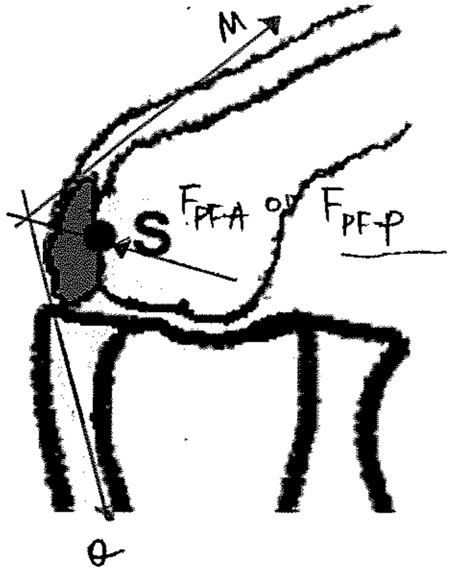
ii) **[10 points]** Continuing from part **(i)**, estimate the ratio of the magnitudes of the patellar-femoral contact force (**FPF-A/FPF-P**) acting at point **S** for these two situations, namely, for having tibial-femoral contact at points A and P from part (i). Assume that point **S** remains unchanged.



——— (5)

Assume: direction of M and Q remain unchanged

According to the three-force triangle the direction of $F_{PF\text{-}A}$ and $F_{PF\text{-}P}$ will be the same.

Then

$$\frac{F_{PF\text{-}A}}{Q_A} = \frac{F_{PF\text{-}P}}{Q_P} \quad \text{(similar triangle)}$$

$$\frac{F_{PF\text{-}A}}{F_{PF\text{-}P}} = \frac{Q_A}{Q_P} = \boxed{3.0}$$

——— (5)

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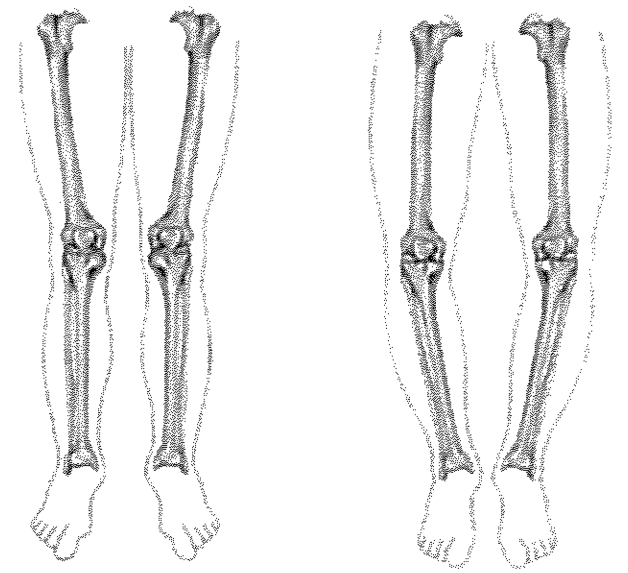
2. [25 points total] Joint Stability
In answering each of the following questions, draw a free-body diagram, perform an equilibrium analysis, and explain your answer.

i) **[10 points]** A bow-legged individual ("varus" alignment) has knee joints that are displaced laterally from normal anatomic position ("normal" alignment), in which the knee joint is directly over the ground reaction force.

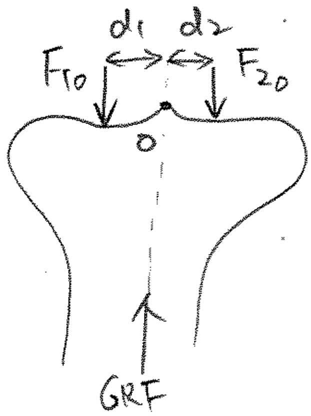
Would you expect the contact force on the medial tibial condyle during standing to be higher or lower for a bow-legged individual compared to someone with normal bone alignment?



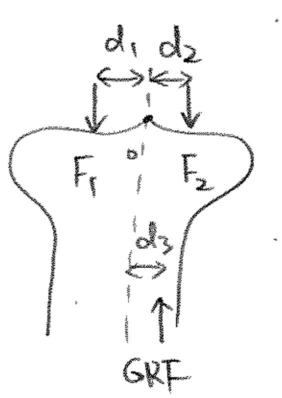
Normal Alignment Varus Alignment

Assume!

- Normal case, GRF pass through O point
- Knee joint is symmetric - - —— - — (3)
- In both cases d_1 & d_2 remain unchanged
- GRF remain unchanged (magnitude & direction)



3) - - -



Normal

y, x, o, $+$

$\Sigma F_x = 0$

$\Sigma F_y = -F_{10} - F_{20} + GRF = 0$

$\Sigma M_o = 0 = F_{10} d_1 - F_{20} d_2$

$\Rightarrow F_{10} = GRF \dfrac{d_2}{d_1 + d_2}$

$F_{20} = GRF \dfrac{d_1}{d_1 + d_2}$

Varus

$\Sigma F_x = 0$

$\Sigma F_y = -F_1 - F_2 + GRF$

$\Sigma M_0 = 0 = F_1 d_1 + GRF d_3 - F_2 d_2$

$\Rightarrow F_1 = GRF \dfrac{d_2 - d_3}{d_1 + d_2}$

$F_2 = GRF \dfrac{d_1 + d_3}{d_1 + d_2}$ - — - - (3)

Conclusion $F_1 < F_{10}$ lateral
$F_2 > F_{20}$ Medial - —— - (1)

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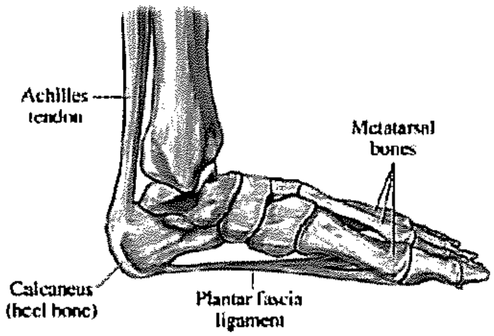
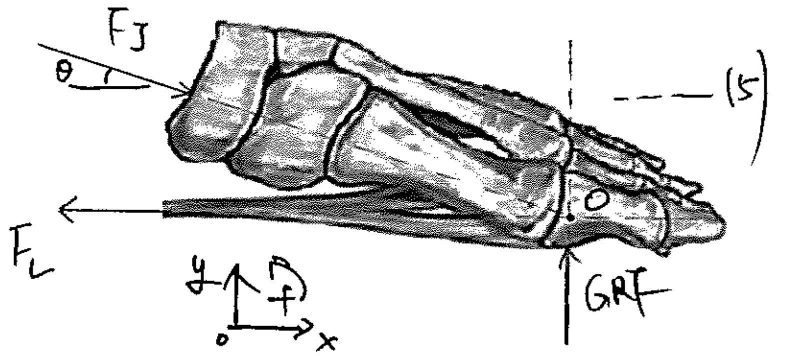
ii) **[15 points]** For clinical problems with plantar fasciitis, the plantar fascia ligament can get damaged due to overloading. Using a free-body analysis of the free body shown on the right, explain how variation in the details of a person's bone anatomy may explain why, *for the same body-weight and external loading*, some people are more likely to get plantar fasciitis than others.





Assume. · F_L is horizontal

· GRF is vertical and magnitude remain unchange

· Three force pass through one point o

$$\Sigma F_x = -F_L + F_J \cdot \cos\theta = 0$$

$$\Sigma F_y = GRF - F_J \sin\theta = 0$$

$$\Rightarrow \frac{GRF}{F_L} = \tan\theta \quad \text{------ (5)}$$

Conclusion: When the bone structure of foot changes, (e.g. flat feet or fallen arches) the θ will be smaller, then $F_L = GRF/\tan\theta$ will increase — (5)

or if the orientation of the bone changes, the contact point will be pushed further away from the ground, which would have the same effect.

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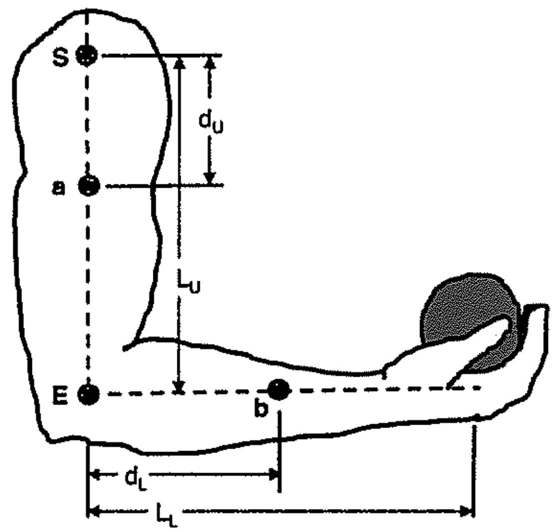
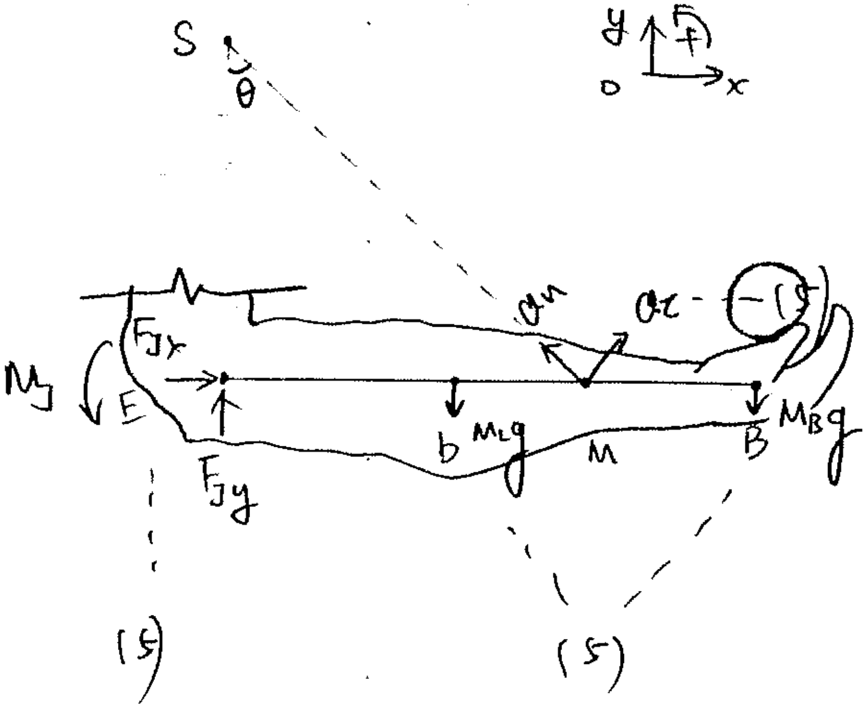
3. [50 points total] Dynamic Analysis

An athlete holds a solid steel ball and then rapidly rotates his arm *counter-clockwise* about the shoulder with an angular velocity of $\omega\ rad/s$ and a *counter-clockwise* angular acceleration of $\alpha\ rad/s^2$ while keeping the elbow and wrist joints rigid. At the given instant, assume the following:

- planar dynamics;
- the point **S** is the instantaneous center of rotation of the rigid arm/hand/ball system;
- the upper arm (mass of M_U) is vertical;
- the forearm and hand (combined mass of M_L) are horizontal, as shown;
- the center of mass of the upper arm is at point **a** and the center of mass of the lower arm (forearm + hand) is at point **b**;
- the steel ball acts as a point mass M_B;
- the mass moment of inertia for the arm segments (about their respective mass centers) are denoted by I_U for the upper arm and by I_L for the forearm including the hand but not the ball.



i) **[15 points]** Draw a *fully labeled* free body diagram, including any accelerations, of the *lower arm-ball combined system* (i.e. from the elbow down), showing the resultant force and moment at the elbow joint. Show your sign convention.



M is the mass center of lower arm - ball system

$\circlearrowleft \alpha \quad a_\tau = \alpha \cdot SM$

$\circlearrowleft \omega \quad a_n = \omega \cdot SM^2$

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ii) **[35 points]** For this planar dynamic problem, write out the three equations of motion for the *lower arm-ball combined system* in terms of *only the quantities* provided in the problem description and the loads and dimensions shown in your free body diagram.

$SE = L_u$

$EM = \dfrac{M_L d_L + M_b L_L}{M_L + M_b}$

$SM = \sqrt{SE^2 + EM^2}$

(10)

$a_\tau = SM \cdot \alpha$

$a_n = SM \cdot \omega^2$

$\sin\theta = \dfrac{EM}{SM}$

$\cos\theta = \dfrac{SE}{SM}$

$I_M = I_L + M_L(EM - d_L)^2 + M_b(L_L - EM)^2 = I_{cm}$

(10)

2D Dynamic Equs

$\Sigma F_x = F_{Jx} = (a_\tau \cos\theta - a_n \sin\theta)(M_L + M_b)$ ——— (5)

$\Sigma F_y = F_{Jy} - (M_L + M_b)g = (a_n \cos\theta + a_\tau \sin\theta)(M_L + M_b)$ —— (5)

$\Sigma M_{cm} = M_J + M_L g(EM - d_L) - M_b g(L_L - EM) - F_{Jy} \cdot EM = I_{cm}\alpha$ —— (5)