Chemical Engineering 150A **April 15, 2015**

Midterm #2

(150) Problem 1. Cone-and-plate viscometer

A cone-and-plate viscometer consists of a stationary flat plate and an inverted cone, whose apex just contacts the plate. The liquid whose viscosity is to be measured is placed in the gap between the cone and plate. The cone is rotated in the ϕ direction at a known angular velocity, Ω, and the torque required to turn the cone, T_z, is measured. The viscosity of the fluid can be expressed in terms of Ω, T_z, and the angle between the cone and plate, ψ_0. For commercial instruments, ψ_0 is very small (about 1°).

The cone-and-plate viscometer is shown in Figure 1 from the side of the instrument (a) and from the top of the instrument (b), showing a differential element $r\ dr\ d\phi$ in spherical coordinates.

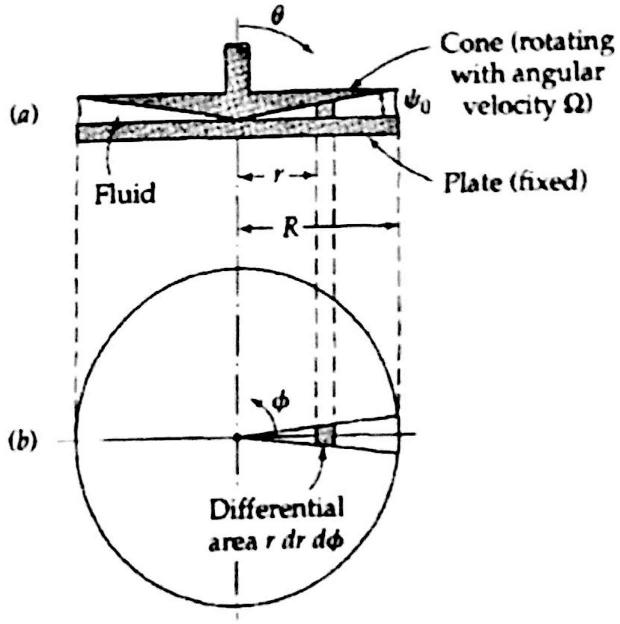

Figure 1. Schematic of a Cone-and-Plate Viscometer

a) **(30)** Using the geometry in Figure 1 (a) and (b), describe the kinematics of flow (i.e., what velocities are nonzero and what spatial variables do they depend on?).

b) **(45)** The fluid in the viscometer is Newtonian and incompressible. Flow is slow and in the creeping regime (i.e., inertial forces are negligible). Gravity is negligible for the dimensions of the viscometer. List the appropriate forms for the equation of continuity and the equations of motion (see below). DO NOT ATTEMPT A SOLUTION. Explain whether the velocity field assumed in part 1 is consistent with what you find from continuity.

Consider now explicitly the case where the angle between the cone and plate, ψ_0, is very small. In this case, the azimuthal flow in Figure 1 can be approximated at each r as local Couette flow, $v_x(y)$, where y is directed from the bottom stationary plate as illustrated in Figure 2. The azimuthal velocity at the top rotating cone is labeled as $V(r)$ in Figure 2 and the local gap thickness is labeled as $b(r)$.

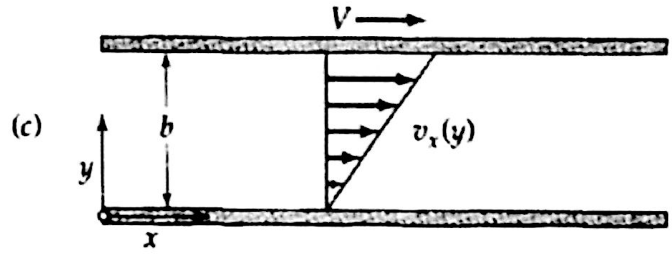

Figure 2. Local Couette Flow in the Cone-and-Plate Viscometer. This figure is a side view of Figure 1 (a) so that the x coordinate in Figure 2 is directed along the ϕ coordinate of Figure 1.

c) **(25)** Let the circular angle from the bottom plate be defined as $\psi = \pi/2 - \theta$ (because ψ_o is very small, so is ψ). Show that the locally linear velocity profile in Figure 2 can be written as

$$v_\varphi(r,\psi) = v_x(y) \sim r\Omega\frac{\psi}{\psi_o} \qquad (1.1)$$

d) **(20)** The definition of shear stress in spherical coordinates is

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] \qquad (1.2)$$

Note that for small ψ_o, $\sin\theta = \sin(\pi/2 - \psi) = \cos\psi \sim 1$. Now from Eq. 1.1, derive an expression for the shear stress exerted by the fluid on the stationary bottom plate.

e) **(30)** From part d), determine an expression for the torque T_z required at the bottom plate to keep it stationary.

(50) Problem 2. Power for California Bullet Train.

The California High-Speed Rail Authority wishes to estimate the size of the power plant required to run the planned high-speed bullet train (near 150 mph). The front of the train is approximately hemispherical of radius R, while the top and sides are long, flat plates of length L. Width of the train is W. Because the train is long, we neglect drag at the front and back. Find an expression for the power required to operate the train at velocity U.

Drag at the bottom (undercarriage) of the train obeys the empirical Blasius correlation (HW 54) with the characteristic distance in the Reynolds number taken as the gap between the undercarriage and the track, H. The drag coefficient for turbulent flow over flat plate of length L is given by integrating Eq. 12-73 of the text over the length of the plate: $C_{DL} = 0.072\,\mathrm{Re}_L^{-1/5}$ where Re_L is the Reynolds number based on the length of the plate.