Chemical Engineering 150A **February 25, 2015**

Midterm #1

(145) 1. Drainage of a Bingham Plastic Fluid from a Vertical Tube

A Bingham plastic is a non-Newtonian fluid that behaves as a rigid body at low stress and flows as a Newtonian fluid at high stress. The material acts as a solid if the magnitude of the viscous stress is less than the yield stress, τ_o. Only when the viscous stress exceeds τ_0 does the material flow continuously as a Newtonian liquid with constant density ρ and viscosity μ.

Suppose a vertical tube of radius R and height h_o is filled with a Bingham plastic. Initially, the bottom of the tube rests on a horizontal plate that prevents flow, while the top is open to the atmosphere (Figure 1a). At time $t = 0$, the bottom plate is removed (Figure 1b). Surface tension is negligible. When flowing, the fluid obeys a pseudo-steady velocity profile

$$v_z(t,r) = v_0(t) \qquad 0 < r < R - \delta$$

$$v_z(t,r) = v_0(t)\frac{R-r}{\delta} \qquad R - \delta < r < R$$

where $v_o(t)$ is the core plug-flow velocity corresponding to where the shear stress is less than τ_o and δ is a small slip length ($\delta/R << 1$). As an approximation, assume that δ is constant.

(25) a. What is the minimum tube radius, R_{min}, required for the fluid to flow when the bottom plate is removed?

(25) b. Consider now (and below) a tube with radius $R = 2R_{min}$ and remove the bottom plate. Find an expression for the shear-stress force exerted by the wall on the fluid. Be sure to label the subscripts on the shear stress.

(50) c. Derive a differential equation describing how long it takes for the tube to empty in terms of the height of the fluid h. Because $\delta/R << 1$, the average velocity is well approximated by $v_o(t)$. Explain the physical meaning of each term in your expression.

(20) d. Characterize the differential equation in part c and list the necessary boundary conditions.

(25) e. Assume that the fluid is quite viscous so that the outflow rate of momentum is comparatively small and the process is pseudo-steady (i.e., momentum accumulation is negligible). Under these approximations, find the time necessary for the tube to drain completely.

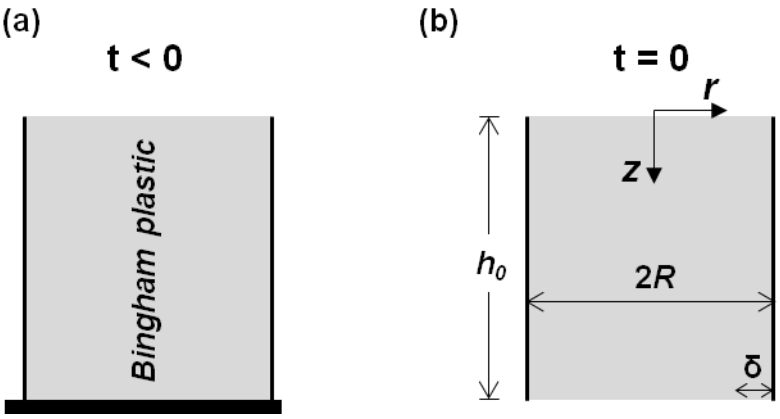

Figure 1. A Bingham plastic fluid (a) confined in a vertical tube and (b) released at time zero

(30) 2. Testing High-Pressure Vessels

What is the most important safety precaution to be taken when testing the strength of a high-pressure vessel with a gas?