Chemical Engineering 150A **April 14, 2008**

Midterm #2

(155) 1. Consider a solid sphere of radius R placed concentrically inside a second sphere of radius aR, as shown in Figure 1. The gap between the two spheres is filled with a Newtonian liquid of constant density, ρ, and viscosity, μ. The inside sphere rotates at a *steady* angular speed ω about the z-axis as shown. The rotation speed and the size of the spheres are small enough that the Reynolds number is much less than unity, $Re=\rho\omega R^2/\mu<<1$. This means that the inertial terms in the equations of motion can be neglected so that the rotation is in the so-called Stokes approximation.

(25) a). Describe the kinematics of the flow. (i.e., which components of velocity are finite and what spatial variables do they depend on.

(20) b). Write the equation of continuity and confirm that your assumed kinematics in part a) are consistent.

(15) c). List the boundary conditions for all of the non-zero velocities in the problem.

(35) d) Starting with the full Navier-Stokes equations in spherical coordinates, write the differential equation for the fluid velocity v_ϕ in the gap between the two spheres. Be sure to remember that inertial terms are to be neglected.

(15) e). It is suggested to assume a product solution for v_ϕ of the form $v_\phi = f(r)sin(\theta)$. Explain why this is a reasonable guess.

(25) f). Substitute the assumed solution in Part e into the φ-component of the equation of motion to obtain a differential equation in terms of $f(r)$. Clearly state the boundary conditions for $f(r)$ and show your work.

(20) g). Solve the *f(r)*-differential equation to obtain v_ϕ. Hint: for an ODE of the form:

$$\sum_i x^i \frac{d^i f}{dx^i} = 0$$

try the substitution $f = x^n$.

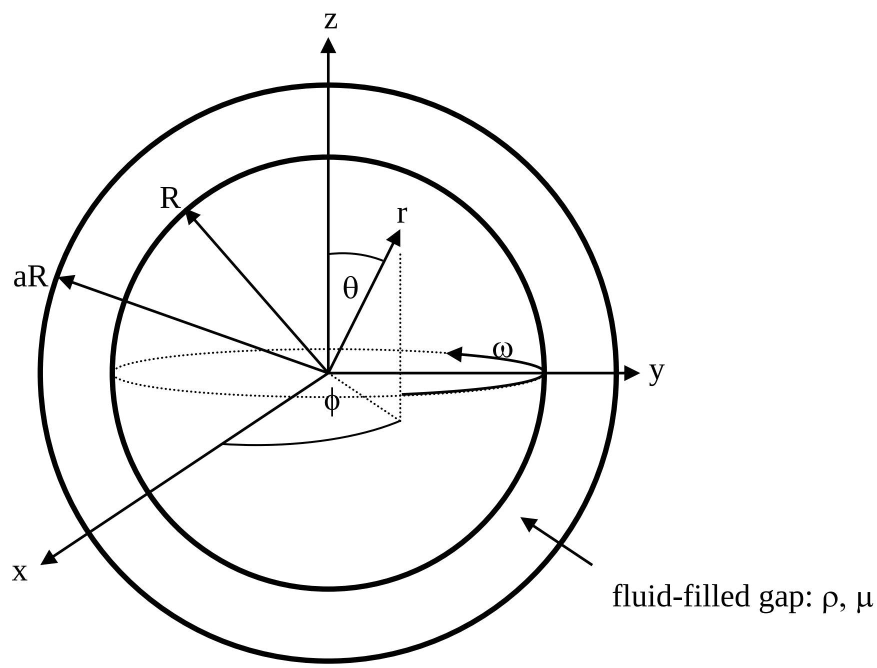

Figure 1: Two concentric spheres of radius *aR* and *R* are separated by an incompressible, Newtonian fluid of constant viscosity. The inner sphere rotates around the z-axis with a steady angular velocity *ω,* while the outer sphere is fixed in place.