Chemical Engineering 150A **February 29, 2008**

Midterm #1

(155) 1. In the system below, a free liquid jet flows downwards at a known volumetric flow rate Q_o against a suspended inverted solid cone of mass m, as shown in Figure 1. A suspending force F_o is applied to the cone to keep it stationary, as illustrated. The liquid deflects around the cone (dimensions are given) with a _local_ x-velocity profile given by

$$v_x(y) = a\frac{\delta^2}{\mu}[1-\left(\frac{y}{\delta}\right)^2]$$

where δ is the thickness of the fluid layer, μ is viscosity of the liquid, and $a = \rho g \cos\beta / 2$ (ρ is the liquid density and g is the acceleration of gravity).

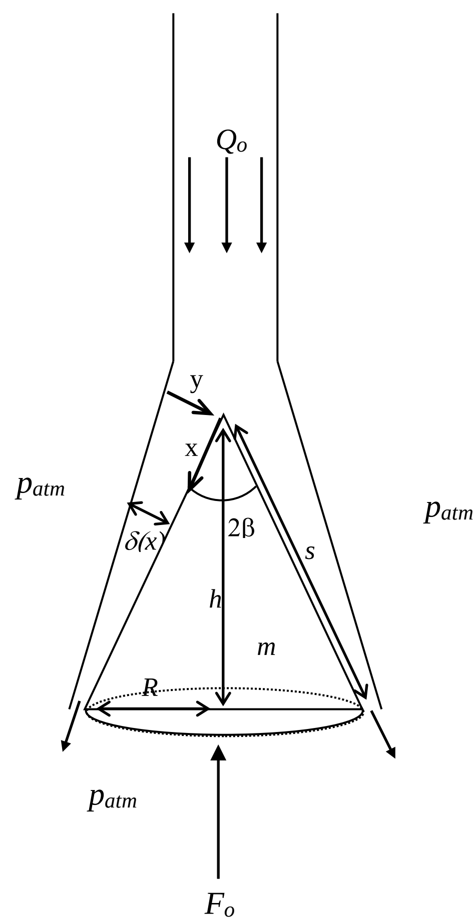

Figure 1. A Free Liquid Jet Impinging on a Suspended Inverted Cone

(40) a. Since the circumference of the cone increasing as the fluid travels along its length *s*, the the liquid film occupies larger cross sectional areas. Hence the film thickness δ decreases with increasing *x*. Perform a mass balance on the fluid to find an expression for the film thickness, δ, as a function of *x* and the parameters as a function of x, Q_o ρ, β, g, and μ.. The volumetric flow of liquid at any point *x* along the cone can be written approximately as

$$Q = 2\pi x\delta \sin\beta \langle v_x \rangle$$

where $\langle v_x \rangle$ is the average *x*-velocity at position *x*.

(15) b. What role does pressure play in keeping the cone stationary?

(15) c. Which components of the shear stress tensor act on the cone (i.e., which of the various τ_{ij})?

(25) d. Find the expression for the vertical component of the drag force exerted by the liquid on the cone in terms of x, Q_o ρ, β, g, and μ. Please note that because the liquid film thickness varies in the *x*-direction along the cone, so does the shear stress.

(40) e. Perform a vertical momentum balance on the cone to establish an expression for the force F_o in terms of the parameters Q_o ρ, β, g, and μ.

(10) f. What will happen if F_o is held constant and the volumetric flow rate of the jet, Q_o, is increased?

(10) g. What happens to the cone if it tips slightly off axis?