Chemical Engineering 150A **February 25, 2005**

Midterm 1

(170) 1. An ideal gas is pumped into a cylinder containing a massless and frictionless piston of diameter, D, through an inlet line with diameter, d, as shown in Figure 1. The inlet gas is at temperature, T, and <u>constant</u> pressure, P_{in}. A counter force, F, is exerted on the piston by means of a spring that obeys Hooke's law: F = kx (k has dimensions of force/ length) where x is the piston displacement. The gas chamber is isothermal at temperature T and of uniform gas properties (i.e., well mixed). In particular, the pressure exerted by the gas on the cylinder walls is everywhere equal to P. Initially, the piston is at rest, and the gas pressure in the cylinder is P_0 ($< P_{in}$).

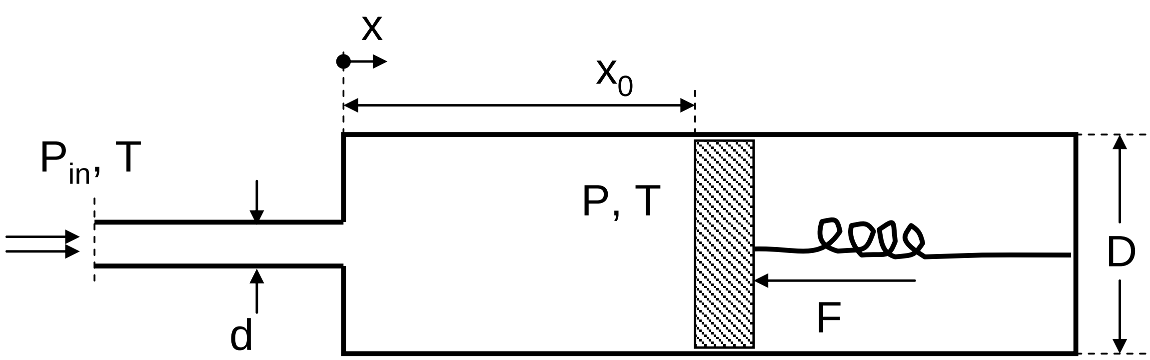

Figure 1. Gas Injection into a Cylinder Containing a Spring Loaded Piston

(20) a. Find an expression for the initial equilibrium location of the piston, x_0.

(20) b. Find an expression for the final equilibrium position of the piston, x_∞.

(10) c. Give the expression for how the gas pressure in the cylinder and the piston displacement are related at any time?

(10) d. Qualitatively, how do you expect the inlet flow rate to vary with time? Provide a simple sketch of this behavior.

(30) e. Perform a transient mass balance on the gas in the cylinder eventually writing it in terms of the gas pressure and the inlet flow rate (and other parameters that you know such as k, d, D, etc).

(30) f. Perform now a transient momentum balance (in the x direction), again in terms of pressure and the inlet flow rate (and other parameters that you know such as k, d, D, etc). Be sure to include all forces on all walls.

(30) g. Eliminate the inlet flow rate between the mass and momentum balances to yield a transient ordinary differential equation for the gas pressure in the cylinder. How many boundary conditions do you need and what are they? Is it possible to integrate analytically this ode? Explain why or why not.

(20) h. From Groucho Marx: Who is buried in Grant's tomb?