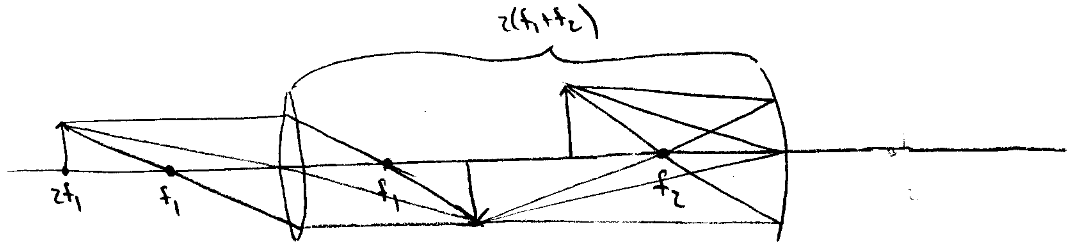
Exam 1: Lee

1.

a)



b) The first image is at

$$i' = \left(\frac{1}{f_1} - \frac{1}{o}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1$$

and so the final image is at

$$i = \left(\frac{1}{f_2} - \frac{1}{2(f_1+f_2)-i'}\right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2}\right)^{-1} = 2f_2$$

The distance of the final image to the lens is then

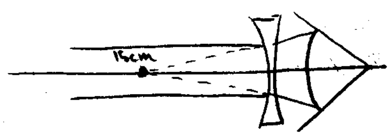
$$2(f_1+f_2) - 2f_2 = 2f_1$$

c) $m = m_{lens} m_{mirror} = \left(\frac{-i'}{o}\right)\left(\frac{-i}{2(f_1+f_2)-i'}\right) = \left(\frac{2f_1}{2f_1}\right)\left(\frac{2f_2}{2f_2}\right) = +1$

d) real – the light focuses to a point

2.

a)



Let the eye-lens distance be $d > 0$, then

$$f = -15 + d$$

b) setting $d = 0$ and noting that the virtual image must be at 10 cm,

$$o = \left(\frac{1}{f} - \frac{1}{i}\right)^{-1} = \left(\frac{1}{-15} - \frac{1}{-10}\right)^{-1} = \left(\frac{1}{10} - \frac{1}{15}\right)^{-1} = 30\,\text{cm}$$

3.

a)

$$\sin\theta = \frac{(m \cancel{+} \frac{1}{2})\lambda}{D} \longrightarrow \frac{(m \cancel{+} \frac{1}{2})\lambda}{2D}$$

Assume $\theta \ll 1$, then

$$\theta \longrightarrow \frac{1}{2}\theta$$

and since $L\tan\theta \approx L\theta = x$, then

$$L \to 2L$$

The dist. to the screen must double

b)

$$E_\theta = E_1 + E_2 = E_0\left[\sin(\omega t) + \sin(\omega t + \delta)\right] = 2E_0 \sin\left(\omega t + \frac{\delta}{2}\right)\cos\left(\frac{\delta}{2}\right)$$

At the center $I = I_0$ and $E_\theta = 2E_0$, where we average over time. Since $I \propto E^2$,

$$\frac{I_\theta}{I_0} = \frac{E_\theta^2}{(2E_0)^2} = \cos^2\left(\frac{\delta}{2}\right)$$

where the time averaging cancels out. δ calculate the phase diff from $d\sin\theta = (m \cancel{+} \frac{1}{2})\lambda$: $\delta/2\pi = d\sin\theta/\lambda$, so

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

c)

$$N = \frac{t}{\lambda/n} - \frac{t}{\lambda} = \frac{t}{\lambda}(n-1)$$

There will be N fringe shifts. For no change N must be an integer:

$$\frac{t}{\lambda} = \frac{N}{n-1}, \quad N \in \mathbb{Z}$$

4

$$A_{bead} \sim V_{bead}^{2/3} = \left(\frac{m}{r}\right)^{2/3} \sim \left(\frac{10^{-6}}{10^{-1}}\right)^{2/3} cm^2 \sim 10^{-3}\, cm^2$$

$$A_{laser} \sim d^2 \sim (10^{-1})^2 = 10^{-2}\, cm^2$$

$\Rightarrow A_{bead} \ll A_{laser}$, SO WE MUST CONSIDER INTENSITY, NOT JUST TOTAL LASERPOWER

$$mg = F = \left(\frac{dp}{dt}\right)_{applied} = \frac{d}{dt}\Big(\underset{\substack{\uparrow \\ \text{perfect} \\ \text{reflection}}}{2} \times E/c\Big)_{applied} = \frac{2}{c} P_{applied}$$

$$= \frac{2}{c} S \cdot \underset{\substack{\uparrow \\ \text{collision} \\ \text{cross-section}}}{\sigma_{bead}} = \frac{2}{c} P \cdot \frac{\sigma_{bead}}{\underset{\nwarrow\ \text{cross-section of laser beam}}{A_{laser}}}$$

$$= \frac{2}{c} P \frac{\pi R^2}{\pi (d/2)^2} = \frac{8}{c} P \frac{R^2}{d^2} \Rightarrow \boxed{P = \frac{mgc}{8}\frac{d^2}{R^2}}$$

$$\frac{m}{r} = \frac{4\pi}{3} R^3 = V_{bead} \Rightarrow R = \left(\frac{3}{4\pi}\frac{m}{r}\right)^{1/3} \simeq 1.17e\text{-}2\, cm$$

$$\rightarrow \boxed{P \simeq 200\, W \quad (\simeq 215.7\, W)}$$