Physics 7B Midterm 1 Solutions - Fall 2017
Professor A. Lanzara

Problem 1

(a) From the ideal gas law

$$P_A V_A = nRT_A \tag{1}$$
$$n = \frac{P_A V_A}{RT_A} \tag{2}$$

(b) For point B, we apply the ideal gas law

$$T_B = \frac{P_B V_B}{nR} = \frac{3P_A V_A}{R} \frac{RT_A}{P_A V_A} \tag{3}$$
$$= 3T_A \tag{4}$$

For point C, we know that $T_A = T_C$ since the two points are connected by an isotherm. Furthermore, this also implies that

$$P_A V_A = P_C V_C \tag{5}$$

Since points B and C are connected by an adiabat, we have

$$P_B V_B^\gamma = 3P_A V_A^\gamma = P_C V_C^\gamma \tag{6}$$

Where $\gamma = 5/3$. Taking the ratio of the two equations, we have

$$3V_A^{\gamma-1} = V_C^{\gamma-1} \tag{7}$$
$$V_C = 3^{\frac{1}{\gamma-1}} V_A \tag{8}$$

and

$$P_C = 3^{\frac{1}{1-\gamma}} P_A \tag{9}$$

(c) We have

$$\Delta U_{AB} = \frac{3}{2}nR(T_B - T_A) = 3P_A V_A \tag{10}$$
$$W_{AB} = 0 \tag{11}$$
$$Q_{AB} = \Delta U_{AB} \tag{12}$$

$$\Delta U_{BC} = \frac{3}{2}nR(T_C - T_B) = -3P_A V_A \tag{13}$$
$$W_{BC} = -\Delta U_{BC} \tag{14}$$
$$Q_{BC} = 0 \tag{15}$$

$$\Delta U_{CA} = 0 \tag{16}$$
$$W_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right) = P_A V_A \ln\left(3^{\frac{1}{1-\gamma}}\right) \tag{17}$$
$$Q_{CA} = W_{CA} \tag{18}$$

(d) Adding all the quantitites for each step,

$$\Delta U = 0 \tag{19}$$

$$Q = 3P_AV_A + P_AV_A \ln\left(3^{\frac{1}{\gamma-1}}\right) \tag{20}$$

$$W = 3P_AV_A + P_AV_A \ln\left(3^{\frac{1}{\gamma-1}}\right) \tag{21}$$

Problem 2

(a) The entropy change for free expansion is given by

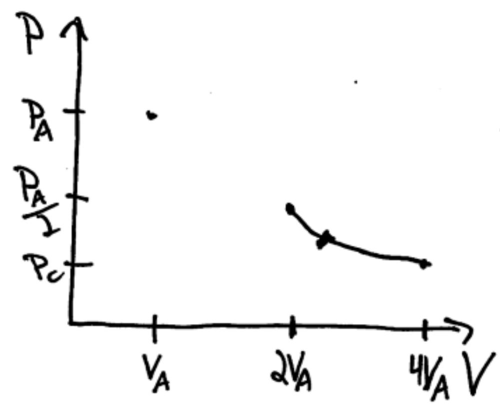
$$\Delta S = \int \frac{dQ}{T} = \int \frac{dW}{T} = \int nR\frac{dV}{V} \tag{22}$$

$$= nR\ln\left(\frac{V_B}{V_A}\right) \tag{23}$$

$$= nR\ln(2) \tag{24}$$

(b) The entropy change for a reversible, adiabatic proccess is zero.

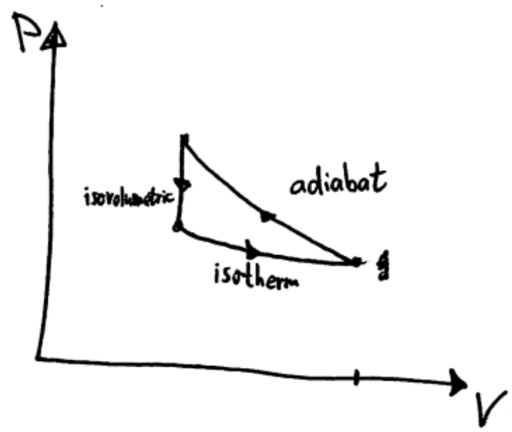
(c) We plot the points on a PV diagram:



(d) No modification since the type of gas does not alter the fact that the change in entropy is zero. Note that the steepness of the curve in part (c) would change due to the change in γ values in going from a diatomic to monotomic ideal gas.

Problem 3

(a) We have the PV diagram:



Using the labels in the above PV diagram,

$$P_0 = \frac{nRT_0}{V_0} \tag{25}$$

$$P_1 = P_0 \frac{V_0^\gamma}{V_1^\gamma} = 3^\gamma P_0 \tag{26}$$
$$T_1 = \frac{P_1 V_1}{nR} = 3^{\gamma-1} \frac{P_0 V_0}{nR} = 3^{\gamma-1} T_0 \tag{27}$$

$$V_2 = V_0/3 \tag{28}$$
$$T_2 = T_0 \tag{29}$$
$$P_2 = \frac{3nRT_0}{V_0} \tag{30}$$

where $\gamma = 5/3$.

(b) The heat needed to melt a mass M of ice is equal to the heat output from the gas on segment $1 \to 2$ minus the heat intake from $2 \to 0$. Since $\Delta U = 0$ for a cycle, we have

$$W_{gas} = Q_{total} = -ML \tag{31}$$

The work done on the gas is the negative of this, so positive work is done on the gas.

Problem 4

(a) The specific heat of the mixture is given by

$$C_m = \frac{C_1 + C_2}{2} \tag{32}$$

so that the heat of the reaction is

$$Q = MC_m \Delta T \tag{33}$$
$$= 2m \frac{C_1 + C_2}{2} (4T_0) \tag{34}$$
$$= 4(C_1 + C_2) m T_0 \tag{35}$$

(b) The beetle's volume goes from V_0 to $V_0 + \Delta V$, where

$$\Delta V = V_0 \beta \Delta T = \beta V_0 T_0 \tag{36}$$

We take the atmosphere to be at constant pressure, so that the work

$$W_{atm} = -P_{atm} \int dV = -P_{atm} \Delta V = -\beta P_{atm} V_0 T_0 \tag{37}$$

Problem 5

(a) From the definition of efficiency

$$\epsilon = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H} \tag{38}$$

we have

$$\epsilon_1 = 1 - \frac{Q_{L1}}{Q_{H1}} \tag{39}$$

$$\epsilon_2 = 1 - \frac{Q_{L2}}{Q_{H2}} \tag{40}$$

since the engines are in series, $Q_{L1} = Q_{H2}$. Treating the two engines as a single engine taking in Q_{H1} and spitting out Q_{L2}, we can define the total, effective efficiency

$$\epsilon_T = 1 - \frac{Q_{L2}}{Q_{H1}} \tag{41}$$

$$= 1 - \frac{1 - \epsilon_1}{Q_{L1}} (Q_{L1} - Q_{L1}\epsilon_2) \tag{42}$$

$$= 1 - (1 - \epsilon_1)(1 - \epsilon_2) \tag{43}$$

$$= \epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2 \tag{44}$$

(b) Using the efficiency of the Carnot engine $\epsilon_c = 1 - \frac{T_L}{T_H}$, we find

$$\epsilon_T = \left(1 - \frac{T_i}{T_h}\right) + \left(1 - \frac{T_c}{T_i}\right) - \left(1 - \frac{T_i}{T_h}\right)\left(1 - \frac{T_c}{T_i}\right) \tag{45}$$

$$= 2 - 1 - \frac{T_i}{T_h} - \frac{T_c}{T_i} + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} \tag{46}$$

$$= 1 - \frac{T_c}{T_h} \tag{47}$$

where we have used

$$\epsilon_1 = 1 - \frac{T_i}{T_h} \tag{48}$$

$$\epsilon_2 = 1 - \frac{T_c}{T_i} \tag{49}$$

(c) The Carnot engine has efficiency given by $\epsilon_c = 1 - \frac{T_L}{T_H}$. Using the defition of efficiency that involves work, $W_1 = W_2$ is equivalent to

$$Q_{H1}\left(1 - \frac{T_i}{T_h}\right) = Q_{H2}\left(1 - \frac{T_c}{T_i}\right) \tag{50}$$

Using the Carnot engine relation $Q_{H1} = \frac{T_h}{T_i}Q_{L1}$ and the fact that $Q_{L1} = Q_{H2}$, the above becomes

$$\frac{T_h}{T_i}\left(1 - \frac{T_i}{T_h}\right) = \left(1 - \frac{T_c}{T_i}\right) \tag{51}$$

which we easily solve with

$$T_i = \frac{T_h + T_c}{2} \tag{52}$$

(d) Using the efficiency of a Carnot engine, $\epsilon_1 = \epsilon_2$ is equivalent to

$$\frac{T_i}{T_h} = \frac{T_c}{T_i} \tag{53}$$

which implies

$$T_i = \sqrt{T_c T_h} \tag{54}$$