Physics 7B Midterm 1 Solutions - Fall 2017
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Problem 1

(a) Let us first calculate the time of heating. From the definition of power, and using the formula relating heat and change in temperature , we have

$$Q = (\text{power})t = mC\Delta T \tag{1}$$

so that

$$t = \frac{mC\Delta T}{(\text{power})} = \frac{1000 \cdot 100}{200}\ \text{s} \tag{2}$$

$$= 500\ \text{s} \tag{3}$$

Now for the coefficient of linear expansion, we apply the linear expansion formula directly and see that

$$\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{0.01}{10 \cdot 100}\,({}^\circ C)^{-1} \tag{4}$$

$$= 10^{-5}\,({}^\circ C)^{-1} \tag{5}$$

(b) For the final equilibrium temperature to be equal to the initial temperature, all of the heat of the rod must go into melting the ice:

$$Q = |m_r C_r (T_f - T_0)| < mL \tag{6}$$

Initially, the ice water mixture is at $0°C$, so

$$m > \frac{1000 \cdot 100}{300000}\ \text{kg} = \frac{1}{3}\ \text{kg} \tag{7}$$

(c) Summing the heat transfers and setting them equal to zero, we have

$$Q_r + Q_w + mL = m_r C_r (T_f - T_{r0}) + (M + m)C_w (T_f - T_{w0}) + mL = 0 \tag{8}$$

$$\tag{9}$$

so that

$$T_f = \frac{(m + M)C_w T_{w0} + m_r C_r T_{r0} - mL}{m_r C_r + (m + M)C_w} \tag{10}$$

$$= 20°C \tag{11}$$

Problem 2

(a) The change in volume is given by the volume expansion formula:

$$\Delta V = V_0 \beta \Delta T = 3V_0 \alpha \Delta T = (1m^3)(3 \times 10^{-3}/°C)(100°C) \tag{12}$$

$$= 0.3m^3 \tag{13}$$

so that the final voume is $V_f = 1.3\ m^3$.

(b) Using the ideal gas law,

$$\begin{aligned} P_f &= \frac{nRT_f}{V_f} && (14) \\ &= \left(\frac{400}{1.3}\right) R \text{ Pa} && (15) \\ &= 300R \text{ Pa} && (16) \end{aligned}$$

Note that we are using just the numerical value of R in the above answer since we have already incorporated its dimensions in the pascal unit.

(c) This is an isovolumetric process, so from the definition of entropy and the first law, we have

$$dS = \frac{dQ}{T} = \frac{1}{T}\left(dU + PdV\right) = C_V n \frac{dT}{T} \tag{17}$$

$$\tag{18}$$

so

$$\begin{aligned} \Delta S &= \frac{5}{2} R \ln\left(\frac{400}{300}\right) \text{ J/K} && (19) \\ &= \frac{5}{2} R \ln\left(\frac{4}{3}\right) \text{ J/K} && (20) \end{aligned}$$

where again we are just using the numerical value of R since we have absorbed its units to get the units of entropy.

Problem 3

(a) If we look at the motion of a gas particle moving only in the x-direction, the time between collisions is given by

$$\Delta t = \frac{2L}{v_x} \tag{21}$$

(b) The average force F_x on one of the walls considered in part (a) is given by

$$F_x = \frac{\Delta \bar{p}_x}{\Delta t} = \frac{2\bar{p}_x \bar{v}_x}{2L} = \frac{mN\overline{v_x^2}}{L} \tag{22}$$

using the fact that the directions are all isotropic, we have $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} + \overline{v_w^2} = 4\overline{v_x^2}$, so

$$F_x = F = \frac{mN\overline{v^2}}{4L} \tag{23}$$

(c) Using the equipartition theorem, we know that

$$K = 2k_B T \tag{24}$$

since each gas particle has 4 degrees of freedom. Using $K = \frac{1}{2}m\bar{v^2}$,

$$\bar{v^2} = 4\frac{k_B T}{m} \tag{25}$$

so that

$$F = \frac{N k_B T}{L} \tag{26}$$

or, equivalently,

$$\frac{F}{L^3}L^4 = P^* Y = N k_B T \tag{27}$$

(d) From the isotropic nature of the dimensions, we know that we can get a count of particle states from

$$\text{number of states} \sim A \exp\left(\frac{-m}{2kT}(v_x^2 + v_y^2 + v_z^2 + v_w^2)\right) dv_x dv_y dv_z dv_w \tag{28}$$

switching to spherical coordinates in 4D, the volume element $dv_x dv_y dv_z dv_w$ becomes proportional to $v^3 dv$, where $v^2 = v_x^2 + v_y^2 + v_z^2 + v_w^2$. You can see this simply from dimensional analysis - the original volume element has dimensions of $(m/s)^4$, so even without knowing how to do spherical coordinates in 4 spatial dimensions we can arrive at this form. Up to dimensionless factors that arise from angular integrals in 4 dimensions, we thus have

$$f(v) \sim \left(\frac{m}{2\pi kT}\right)^2 v^3 e^{-\frac{mv^2}{2kT}} \tag{29}$$

Problem 4

(a) For Segment 1:

$$\Delta U_1 = 0 \tag{30}$$
$$W_1 = nRT_a \ln\left(\frac{V_2}{V_1}\right) = P_a V_1 \ln\left(\frac{V_2}{V_1}\right) \tag{31}$$
$$Q_1 = W_1 \tag{32}$$

For Segment 2:

$$\Delta U_2 = \frac{5}{2}nR(T_2 - T_1) = \frac{5}{2}(P_c V_2 - P_a V_1) \tag{33}$$
$$W_2 = 0 \tag{34}$$
$$Q_2 = \Delta U_2 \tag{35}$$

For Segment 3:

$$\Delta U_3 = 0 \tag{36}$$
$$W_3 = nRT_2 \ln\left(\frac{V_1}{V_2}\right) = P_c V_2 \ln\left(\frac{V_1}{V_2}\right) \tag{37}$$
$$Q_3 = W_3 \tag{38}$$
$$\tag{39}$$

For Segment 4:

$$\Delta U_4 = \frac{5}{2} nR(T_1 - T_2) = \frac{5}{2}(P_a V_1 - P_c V_2) \tag{40}$$
$$W_4 = 0 \tag{41}$$
$$Q_4 = \Delta U_4 \tag{42}$$

(b) From the definition of efficiency,

$$\epsilon = \frac{W_1 + W_3}{Q_1 + Q_4} \tag{43}$$
$$= \frac{P_a V_1 \ln\left(\frac{V_2}{V_1}\right) + P_c V_2 \ln\left(\frac{V_1}{V_2}\right)}{P_a V_1 \ln\left(\frac{V_2}{V_1}\right) + \frac{5}{2}(P_a V_1 - P_c V_2)} \tag{44}$$

(c) Since the Carnot engine is the most efficient engine, and our engine is not the Carnot engine, our engine must be less efficient than the Carnot engine.

(d) Switching to a monatomic gas amounts to changing the $\frac{5}{2}$ in the denominator of ϵ to $\frac{3}{2}$. Keeping everything else constant, this would increase the efficiency.

Problem 5

(a) For an adiabatic process, Q = 0, so first law tells us that

$$dU = -PdV = nC_V dT \tag{45}$$

or

$$PdV + nC_V dT = 0 \tag{46}$$

Where we have used the expression $dU = nC_V dT$, which holds for an ideal gas. Next, from the ideal gas law, we take a variation of all variables (but fixed particle number) to obtain

$$PdV + VdP = nRdT \tag{47}$$

plugging this definition of $nRdT$ into our equation from the first law, we get

$$0 = nC_V \left(\frac{PdV + VdP}{nR} \right) + PdV \tag{48}$$

$$= (C_V + R)PdV + C_V VdP \tag{49}$$

$$= C_P PdV + C_V dP \tag{50}$$

$$= \frac{dP}{P} + \gamma \frac{dV}{V} \tag{51}$$

where we have used $C_P = C_V + R$. Integrating, we have

$$\ln \left(\frac{P_2}{P_1} \right) = \ln \left(\frac{V_1^\gamma}{V_2^\gamma} \right) \tag{52}$$

exponentiating both sides,

$$P_1 V_1^\gamma = P_2 V_2^\gamma \tag{53}$$

(b) We replace the process in the diagram with an isothermal process so that

$$dS = \frac{dQ}{T} = \frac{dU + dW}{T} = \frac{PdV}{T} = \frac{nRdV}{V} \tag{54}$$

integrating , we find

$$\Delta S = nR \ln \left(\frac{V_b}{V_a} \right) \tag{55}$$