Physics 7B Midterm 1 - Fall 2017
Professor R. Birgeneau

Total Points: 100 (5 Problems) + 5 Bonus

This exam is out of 100 points with 5 bonus points avaliable. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. **Good luck!**

Problem 1 (20 pts.)

(a) (7 pts.) A metal rod of mass 1 kg, specific heat 1000 J/kg °C, initial length 10 cm, and initial temperature 20 °C is placed on a hot plate. Heat flows into the rod with a power of 200 W. After a time t, the metal rod has reached a temperature of 120 °C and now has a length of 10.01 cm. Determine the time t and the coefficient of linear expansion α.

(b) (7 pts.) The rod is cooled to 100 °C and is then immersed in a mixture of ice and water initially in equilibrium. Initially, the ice has mass m and the water has mass M. Find a condition on m that will ensure the final equilibrium temperature T_f of the system is unchanged from the initial temperature of the ice+water mixture. (For simplicity, you may approximate the specific heat of water as 5000 J/kg°C and its heat of fusion as 300000 J/kg, should these quantities be necessary. Also, assume that $P = 1$ atm.)

(c) (6 pts.) Let $m = 0.1$ kg and $M = 0.4$ kg. Determine the final equilibrium temperature T_f (°C) of the system.

Problem 2 (20 pts.)

A container of volume 1 m^3 and coefficient of linear expansion $\alpha = 1 \times 10^{-3}$/°C is filled with 1 mol of CO_2 at 300 K. Heat is added to the system until the temperature reaches 400 K. Treat CO_2 as a gas with 5 degrees of freedom.

(a) (7 pts.) What is the new volume of the container?

(b) (7 pts.) What is the final pressure of the system, in terms of R?

(c) (6 pts.) Assume that $\alpha = 0$. What is the change in entropy of the system in this process?

Problem 3 (25 pts.)

Consider (but do not attempt to picture) a gas of N particles living in **four** isotropic spatial dimensions so that we label the position of a particle at a moment of time as a list of coodinates (x,y,z,w). We will also contain the gas of particles inside of a tesseract (the 4D generalization of a cube) with side lengths l and 4D "volume" $Y = l^4$. Assume that the particles do not interact with each other and that they collide elastically with the tesseract walls.

(a) (4 pts.) How much time is there between collisions of a particle on a given wall?

(b) (8 pts.) Find an expression for the force F exerted on a given wall by the collision of gas particles.

(c) (8 pts.) Using the equipartition theorem, derive the equation of state of the gas that relates the temperature T, the 4D volume Y, and the "pressure" $P^* = F/l^3$.

(d) Bonus (5 pts.): The Maxwell distribution in three spatial dimensions is given, as a proportionality, by

$$f(v) \sim \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}$$

What would the equivalent expression be for the Maxwell distribution in four spatial dimensions?

Problem 4 (20 pts.)

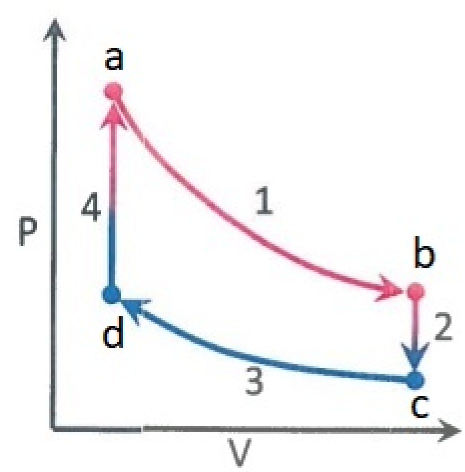
The Stirling Cycle (as shown on the following page) consists of four segments, labeled 1: isothermal, 2: isovolumetric, 3: isothermal, 4: isovolumetric. Given that the top left corner has pressure P_a and volume V_1, and the bottom right corner is at pressure P_c and volume V_2, and that the gas is a diatomic ideal gas with n moles (include only translational and rotational degrees of freedom), answer the following:

(a) (7 pts.) Determine the work done and the heat absorbed by the gas in segments 1,2,3, and 4?

(b) (7 pts.) Calculate the efficiency of this engine using the definition $\epsilon = \frac{W}{Q_H}$, where W is the work done by the gas in a full cycle and Q_H is the total heat flowing into the gas.

(c) (3 pts.) Is the efficiency calculated above greater than, equal to, or less than the efficiency of the Carnot cycle? No mathematical proof is necessary.

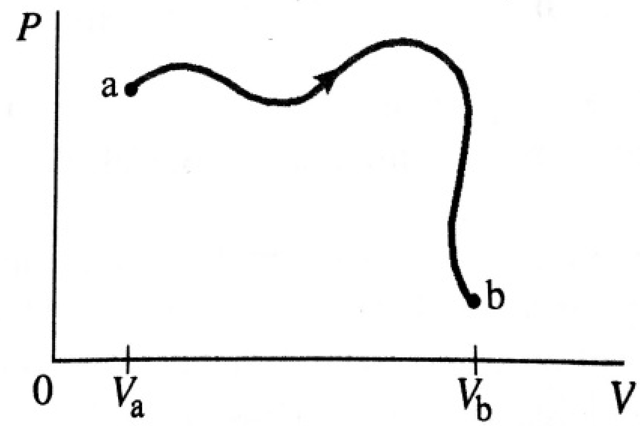
(d) (3 pts.) If we substituted a monatomic gas, would the efficiency increase, decrease, or remain the same? Explain briefly.



Problem 5 (20 pts.)

(a) (15 pts.) Starting from the first law of thermodynamics and the ideal gas law, show that $PV^\gamma =$ constant for adiabatic processes, where $\gamma = C_P/C_V$.

(b) (5 pts.) An ideal gas of n moles undergoes the reversible process ab shown in the PV diagram below. The temperature T of the gas is the same at points a and b. Determine the change in entropy of the gas due to this process.



Formula Sheet: Physics 7B, Midterm 1

Thermodynamics

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$Q = mc\Delta T = nC\Delta T$$

$$C_P - C_V = R = N_A k_B$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} \text{ (for a monatomic gas)}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$$\Delta S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0$$

$$\oint dS = 0$$