Solutions ______, ______, ______
Last First Initial
Name (please print)

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Student Number

UNIVERSITY OF CALIFORNIA AT BERKELEY
DEPARTMENT OF PHYSICS
PHYSICS 7C
FALL SEMESTER 2008
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Midterm #2 November 4, 2008

This examination is closed book. You may refer to a single $8\frac{1}{2}\times 11$ sheet of paper (both sides) that you have prepared. You may use a calculator. Laptops and CellPhones are prohibited..

Work all problems.

Please use the following approximate values:
- Electron mass = 500 keV/c^2
- Proton, antiproton and neutron masses = 1 GeV/c^2
- Pion mass = 140 MeV/c^2

Some of the following constants may be of use.

- hc =1240 eV-nm
- $\hbar$c = 197 eV-nm
- k = $1/4\pi\varepsilon_0$ = 9 x 10^9
- $e^2 / 4\pi\varepsilon_0$ = 1.44 eV-nm
- c = 1 foot / ns

1	______/15
2	______/20
3	______/25
4	______/35
Total	______/95

1. (15 points) For each of the following four-vectors indicate, in the space provided, whether it is time-like, light-like or space-like.

The vector for the interval between each the following pairs of events:

(a) A radioactive particle created at a point and the decay of the same particle 300 meters away 2×10^{-6} seconds later.

time /2

(b) A radioactive particle created at a point and the decay of another particle 300 meters away 2×10^{-8} seconds later.

space /2

(c) John in New York placing a call to Boston at exactly 12:00 and Mary answering a phone in Boston at exactly 12:00. Note Boston and New York are in the same time zone.

space /2

(d) Mary placing a call to the John from Boston at 1:00 and John's phone ringing in New York in response to her call.

time /2

(e) The four-momentum an electron.

time /2

(f) The four- momentum of a photon emitted from a hydrogen atom that is moving with a β =4/5c evaluated in the rest frame of the moving atom.

light /2

(g) A particle of mass m decays into two particles of equal masses, m′. Consider the difference in the four-momentum of the 2 decay particles. Is it time or space like?

space /3

2 (25 points) A rocket ship is traveling in the plus x direction relative to earth observers. It has a speed of $\beta = \frac{4}{5}$ thus $\gamma = \frac{5}{3}$ and $\beta\gamma = \frac{4}{3}$. Bill is standing at the rear of the ship and Betty is standing at the front a distance ℓ_0 from Bill. On the earth, we observe Bill and Betty smile at each other at the same time.

a. How far apart are Bill and Betty as observed in the earth frame?
b. In the frame of the rocket, who smiled first?
c. By how much?

a. l_0 is a proper length in the rocket frame.
∴ In earth frame $l = \frac{l_0}{\gamma} = \frac{3}{5} l_0$

b. In the lab frame the events are simultaneous.
at $ct = 0$ at $x_{Bill} = \frac{-l_0}{2\gamma}$ $x_{Betty} = \frac{l_0}{2\gamma}$

∴ In rocket frame Betty $(ct') = (0) - \beta\gamma x = \frac{-\beta l_0}{2} = -\frac{2}{5} l_0$

Bill $(ct') = (0) - \beta\gamma x = +\frac{\beta l_0}{2} = \frac{2}{5} l_0$

$\Delta ct' = \frac{4}{5} l_0$

Betty is first. @ $t = -\frac{2}{5} l_0$

3.(25 points)

a) The work function for an electron in DNA is about 1 eV. An incoming UV ray of 50nm scatters off the electron and ionizes the DNA (frees the electron). After the scattering, the electron is detected to have a kinetic energy of 3eV. What is the wavelength of the scattered ray?

b) An x-ray of 1000 keV scatters off a free electron that is at rest. If the scattered x-ray has energy of 500 keV, what is the direction (angle relative to incoming x-ray) of the scattered photon?

c) What is the energy and direction (angle relative to the incoming x-ray) of the scattered electron (from part b)?

a) One way: Energy of incident $\gamma = \frac{1240}{50} = 24.8$ eV.

Total Energy transferred to electron $E = 3+1 = 4$ eV.

Energy of scattered $\gamma_{scat} = 20.8$ eV

$\lambda_{scat} = \frac{1240}{20.8} = 59.6$ nm

b) 

One way: $\underset{\sim}{P_{\gamma_1}} + \underset{\sim}{P_T} = \underset{\sim}{P_{\gamma_2}} + \underset{\sim}{P_e}$

$\underset{\sim}{P_e} = \underset{\sim}{P_{\gamma_1}} - \underset{\sim}{P_{\gamma_2}} + \underset{\sim}{P_T}$ so $m^2 = 0 + 0 + m^2 + 2(-\underset{\sim}{P_{\gamma_1}}\cdot\underset{\sim}{P_{\gamma_2}} + \underset{\sim}{P_{\gamma_1}}\cdot\underset{\sim}{P_T} - \underset{\sim}{P_{\gamma_2}}\,\underset{\sim}{P_T})$

where $\underset{\sim}{P_{\gamma_1}}\cdot\underset{\sim}{P_T} = E_{\gamma_1} m$ and $\underset{\sim}{P_{\gamma_2}}\,\underset{\sim}{P_T} = E_{\gamma_2} m$

so

$m(E_{\gamma_1} - E_{\gamma_2}) = E_{\gamma_1}E_{\gamma_2}(1 - \cos\theta_\gamma)$

(you could derive this from Compton scattering formula)

$1 - \cos\theta_\gamma = m\left(\frac{E_{\gamma_1} - E_{\gamma_2}}{E_{\gamma_1} E_{\gamma_2}}\right) = \frac{500 \times 500}{500 \times 1000} = \frac{1}{2}, \quad \cos\theta = \frac{1}{2}$

$\underline{\theta_\gamma = 60^\circ}$

c) Energy of the electron $E_e = E_{\gamma_1} + E_T - E_{\gamma_2} = 1000$ keV

$P_e = \sqrt{(1000)^2 - (500)^2} = 866$ keV/c

$P_{\gamma_2} \sin\theta_{\gamma_2} = P_e \sin\theta_E, \quad \sin\theta_e = \frac{500}{866}\sin 60^\circ = .5$

$\therefore \underline{\theta_e = 30^\circ}$

4. (35 points) A proton of total energy E strikes an antiproton at rest. The two particles stick together forming a single particle of mass M we will call 'X'. The proton and antiproton have mass m.

a. In terms m and M find an expression for E.

b. What is the value of E for m=1 Gev/c^2 and M=3 Gev/c^2.

c. Calculate the value of the total energy of particle X.

d. Calculate $|p_x|$ the value of the magnitude of the momentum of X.

e. Calculate $|p_0|$ the value of the magnitude of the momentum of the incident proton.

f. What is the relation between $|p_x|$ and $|p_0|$?

One Way

a) Invariant mass of initial state $(P_I + P_T)^2 = 2m^2 + 2P_I \cdot P_T$

$= 2m^2 + 2E_I m$

Invariant mass of final state $= M^2$

$$\therefore \quad E_I = \frac{M^2 - 2m^2}{2m}$$

b) $E_I = \frac{3^2 - 2}{2} = \frac{7}{2}$ GeV

c) $E_x = E_I + E_T = \frac{7}{2} + 1 = \frac{9}{2}$ GeV

d) $P_x = \sqrt{E_x^2 - M^2} = \sqrt{(\frac{9}{2})^2 - 3^2} = \sqrt{\frac{45}{4}} = 3.35$ GeV/c

e) $P_I = \sqrt{E_I^2 - m^2} = \sqrt{(\frac{7}{2})^2 - 1} = \sqrt{\frac{45}{4}} = 3.35$ GeV/c

f) They are the same due to conservation of momentum.

See Next Page for another way.

4. (35 points) A proton of total energy E strikes an antiproton at rest. The two particles stick together forming a single particle of mass M we will call 'X'. The proton and antiproton have mass m. (Neglect the coulomb interaction)

a. In terms m and M find an expression for E.

For parts b through f, $m=1$ GeV/c^2 and $M=3$ GeV/c^2.

b. What is the value of E .

c. Calculate the value of the total energy of particle X.

d. Calculate $|p_x|$ the value of the magnitude of the momentum of X. (Give answer in GeV/c)

e. Calculate $|p_0|$ the value of the magnitude of the momentum of the incident proton. (Give answer in GeV/c)

f. What is the relation between $|p_x|$ and $|p_0|$? Why?

Another Way.

Conservation of Energy & momentum

a) Conservation of Energy $E_I + m = E_X$

" of Momentum $P_I = P_X \Rightarrow E_I^2 - m^2 = E_X^2 - M^2$

So. $(E_I + m)^2 = E_I^2 - m^2 + M^2$

Or $\cancel{E_I^2} + 2E_I m + m^2 = \cancel{E_I^2} - m^2 + M^2$

$$E_I = \frac{M^2 - 2m^2}{2m}$$

The rest is the same.