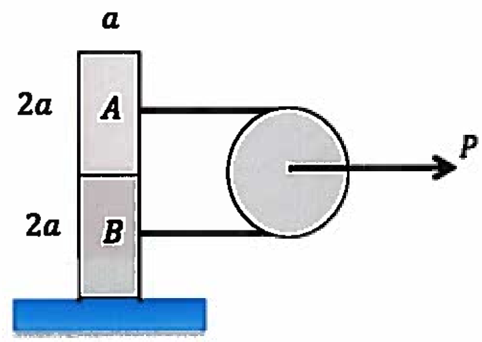
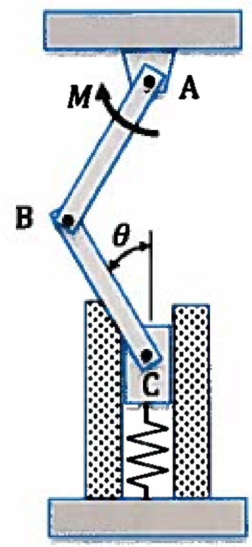
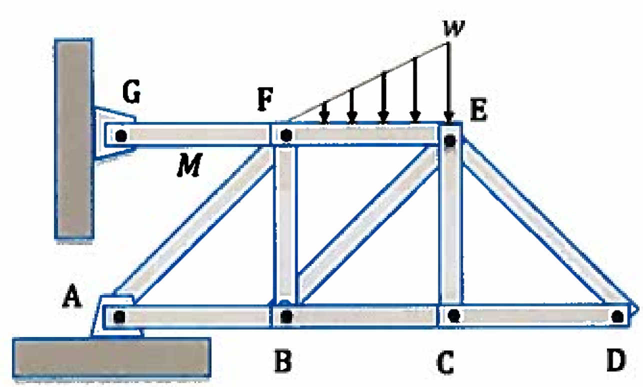
Problem 1. (35 points)
Two stacked blocks, each of width a and height $2a$, are pulled by cables that pass around an ideal pulley. The cables are attached at the vertical center of each block. The blocks are made of different materials so that the top block, A, has mass of m while the lower block, B, has mass of $2m$. The coefficients of friction between the two blocks and between the lower block and the ground are both $\mu_s = 0.5$. *Determine the maximum force P that can be applied to the pulley without motion occurring.* Be sure to identify which block is about to slip or tip at the point of impending motion.



Problem 2. (35 points)
Piston C moves freely in the vertical direction, but is constrained against motion in the horizontal direction by smooth walls. The piston is attached at the bottom to a spring with stiffness k. The spring is unstretched when $\theta = 0$. Members AB and BC are each of length L and may be treated as massless. *Determine the couple moment M required to maintain the mechanism in the position shown.*



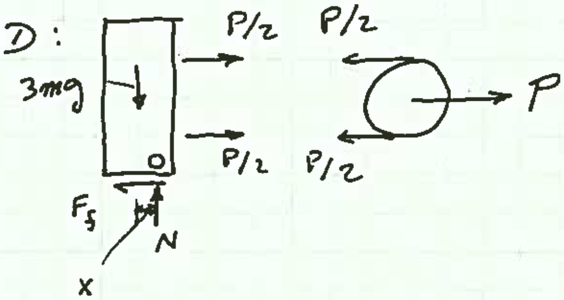
Problem 3. (30 points)
The truss-like frame shown supports a distributed load along member FE that increases linearly from F to E, reaching a maximum force per unit length of w at joint E. All horizontal and vertical members are of length L and all members are connected by pins. *Determine the forces in members* AB, BC *and* AF.



1. THIS SOLUTION WILL FIRST CONSIDER THE MOTION OF BOTH BLOCKS, I.E. SLIPPING OR TIPPING OF BLOCK B ON THE GROUND.

ASSUME STATIC EQUILIBRIUM AND EXAMINE IMPENDING MOTION

FBD:



$$\Sigma F_y = 0 = N - 3mg \Rightarrow N = 3mg$$

$$\Sigma F_x = 0 = P - F_f \Rightarrow F_f = P$$

BUT $F_f \leq \mu N$

$$\Sigma M_o = 0 = x(3mg) - a\left(\frac{P}{2}\right) - 3a\left(\frac{P}{2}\right) \Rightarrow$$

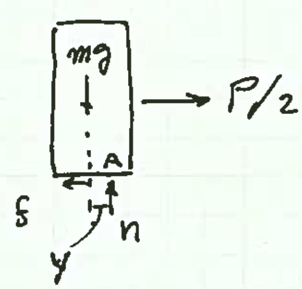
$$x = \frac{2Pa}{3mg}$$

SLIP IF $F_f = \mu N = 3\mu mg \underset{\mu = 0.5}{=} \boxed{\frac{3}{2}mg = P_1}$

TIP IF $x = \frac{a}{2} = \frac{2Pa}{3mg} \Rightarrow \boxed{P_2 = \frac{3mg}{4}} < P_1$

SO IF THE BOTTOM BLOCK MOVES, IT WILL TIP. NOW LET'S SEE IF THE TOP BLOCK MOVES FIRST.

FBD



$$\Sigma F_y = 0 \Rightarrow n = mg$$

$$\Sigma F_x = 0 \Rightarrow f = P/2 \leq \mu n$$

$$\Sigma M_A = 0 = y(mg) - a\left(\frac{P}{2}\right)$$

$$\Rightarrow y = \frac{Pa}{2mg}$$

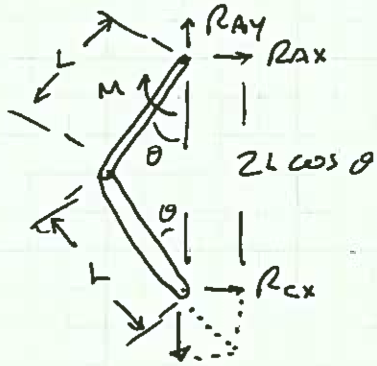
SLIP: $f = \mu n \Rightarrow \frac{P}{2} = \mu mg$

$\mu = 0.5$: $\boxed{P_3 = 2\mu mg = mg}$

TIP: $y = \frac{a}{2} = \frac{Pa}{2mg} \Rightarrow \boxed{P_4 = mg}$

OF THE 4 OPTIONS, P_2 IS THE SMALLEST, SO THE BOTTOM BLOCK WILL TIP AT $P = 3mg/4$

2. NOTE: MEMBER BC IS A 2-FORCE MEMBER.



LET δ BE THE AMOUNT THAT THE SPRING STRETCHES

$$\delta = 2L - 2L\cos\theta$$

$$\Sigma M_A = 0 = -M + (2L\cos\theta)R_{CX}$$

$$k\delta = R_{CY} = 2kL(1-\cos\theta)$$

$$R_{CX} = \frac{M}{2L\cos\theta}$$

SINCE BC IS A 2-FORCE MEMBER, WE KNOW THE DIRECTION OF THE REACTION FORCE AT C SO

$$\tan\theta = \frac{R_{CX}}{R_{CY}} = \left(\frac{M}{2L\cos\theta}\right) \Big/ \left(2kL(1-\cos\theta)\right)$$

$$M = 4kL^2\cos\theta\tan\theta(1-\cos\theta)$$

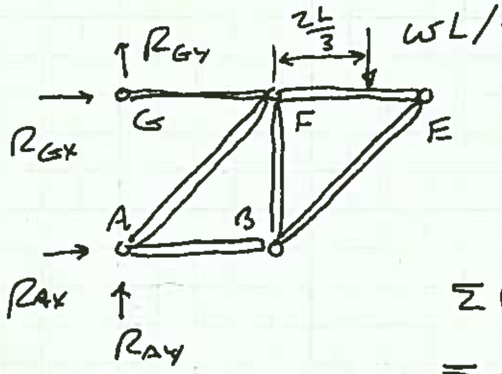
$$\boxed{M = 4kL^2\sin\theta(1-\cos\theta)}$$

3.

BEGIN BY NOTING THAT D IS "FREE" AND HAS ONLY 2 NON-ALIGNED MEMBERS $\Rightarrow$ CD + DE ARE ZERO-FORCE MEMBERS. SIMILARLY CE IS A ZERO-FORCE MEMBER AND SO IS BC $\Rightarrow$

$$\boxed{F_{BC} = 0}$$

NOW DRAW FBD OF THE REST OF THE FRAME



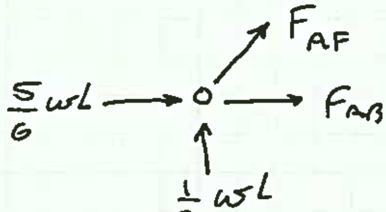
SINCE GF IS A 2-FORCE MEMBER, $R_{Gy} = 0$

$$\Sigma F_y = 0 \Rightarrow R_{Ay} = wL/2$$

$$\Sigma M_G = 0 = -\left(\frac{5L}{3}\right)\left(\frac{wL}{2}\right) + L R_{Ax}$$

$$R_{Ax} = \frac{5}{6} wL$$

JOINT A



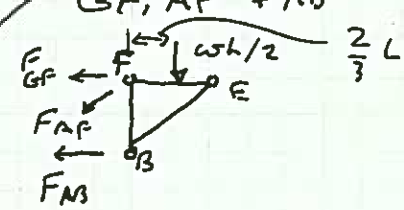
$$\Sigma F_y = 0 = F_{AF}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}wL$$

$$\boxed{F_{AF} = -\frac{wL}{\sqrt{2}}}$$

$$\Sigma F_x = 0 = \frac{5}{6}wL + F_{AB} - \left(\frac{wL}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{F_{AB} = -\frac{1}{3}wL}$$

ALSO, METHOD OF SECTIONS CAN BE USED "CUTTING" THRU GF, AF + AB



$$\Sigma M_B = 0 = -\left(\frac{2L}{3}\right)\left(\frac{wL}{2}\right) - F_{AB}(L) \Rightarrow$$

$$F_{AB} = -\frac{wL}{3}$$