EECS C128/ ME C134
Final
Fri. Dec. 18, 2015
1910-2200 pm

Name:____________________________
SID:_____________

- Closed book. One page, 2 sides of formula sheets. No calculators.
- There are 6 problems worth 100 points total.

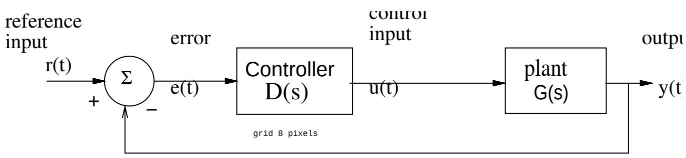
Problem	Points	Score
1	15	
2	16	
3	18	
4	20	
5	16	
6	15	
Total	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of 'F' and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

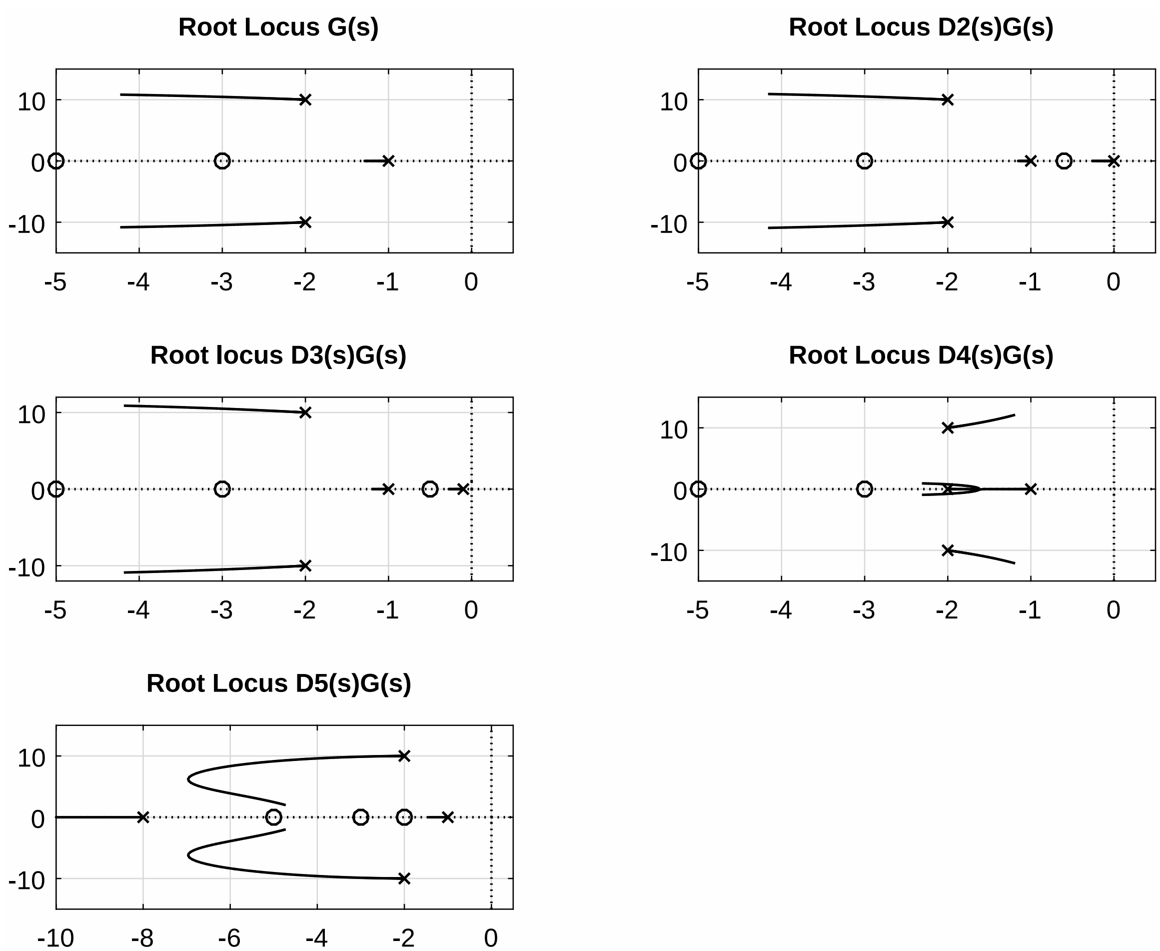
Problem 1 (15 pts)



You are given the open-loop plant:

$$G(s) = \frac{5(s+5)(s+3)}{(s+1)(s^2+4s+104)}.$$

For the above system, the partial root locus is shown for 5 different controller/plant combinations, $G(s), D_2(s)G(s), ..., D_5(s)G(s)$. (Note: the root locus shows open-loop pole locations for $D(s)G(s)$, and closed-loop poles for $\frac{DG}{1+DG}$ are at end points of branches).



[5 pts] a) For each set of open-loop poles and zeros given above, choose the best corresponding open-loop Bode plot V,W,X,Y, or Z from the next page:

(i) $G(s)$: Bode Plot ____

(ii) $D_2(s)G(s)$: Bode plot ____

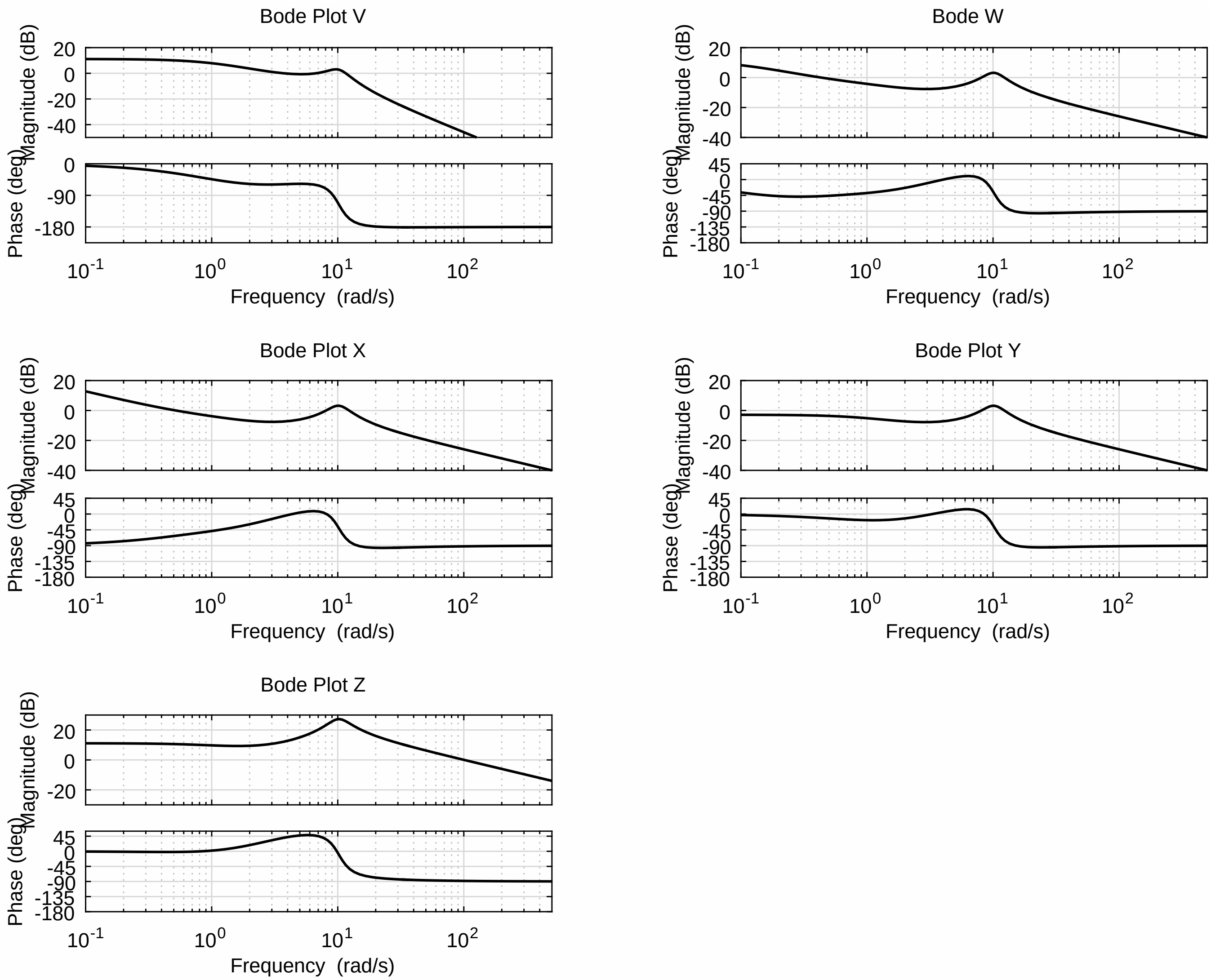
(iii) $D_3(s)G(s)$: Bode plot ____

(iv) $D_4(s)G(s)$: Bode Plot ____

(v) $D_5(s)G(s)$: Bode Plot ____

Problem 1, cont.

The open-loop Bode plots for 5 different controller/plant combinations, $D_1(s)G(s), ..., D_5(s)G(s)$ are shown below.



[5 pts] b) For the Bode plots above:

(i) Bode plot V: phase margin ____ (degrees) at $\omega =$ ____
Bode plot V: gain margin ___ dB at $\omega =$ ____
Estimate damping factor $\zeta =$ ___

(ii) Bode plot Z: phase margin ____ (degrees) at $\omega =$ ____
Bode plot Z: gain margin ___ dB at $\omega =$ ____

Problem 1, cont.

[5 pts] c) For each closed loop controller/plant with root locus as given in part a), choose the best corresponding closed-loop step response (A-E). (Note: dashed line shows final value.)

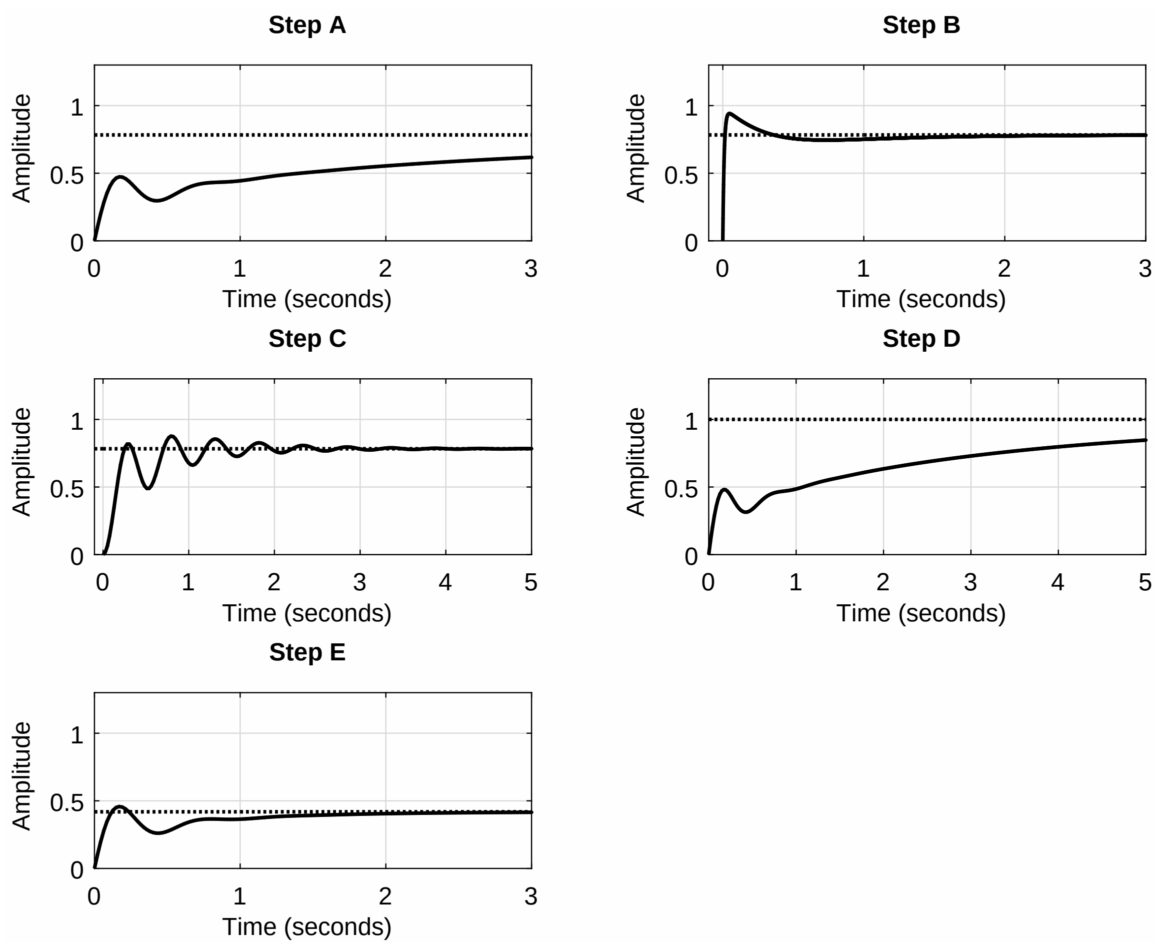
(i) $G(s)$: step response ____

(ii) $D_2(s)G(s)$: step response ____

(iii) $D_3(s)G(s)$: step response ____

(iv) $D_4(s)G(s)$: step response ____

(v) $D_5(s)G(s)$: step response ____



Problem 2 (16 pts)

The open-loop system is given by $G(s) = \frac{400}{(s+2)^2(s^2+2s+101)}$, and Bode plot for $G(s)$ is here:

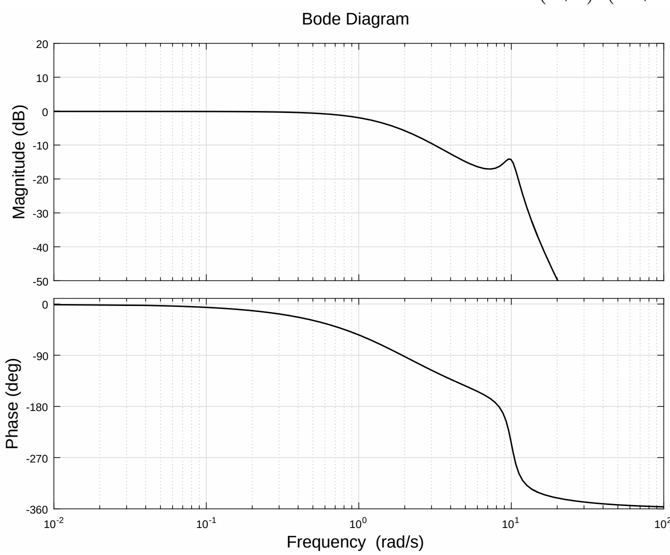


Fig. 3.1

A lag controller $D(s) = k\frac{s+\alpha}{s+\beta}$ is to be designed such that the unity gain feedback system with openloop transfer function $D(s)G(s)$ has static error constant $K_p = 10$. $D(s)G(s)$ should have a nominal (asymptotic approximation) phase margin $\phi_m \approx 40°$ at $\omega_{pm} = 2$ rad s^{-1}.

[6 pts] a. Determine gain, zero, and pole location for the lag network $D(s)$:

gain $k =$ ____ zero: $\alpha =$ ____ pole: $\beta =$ ____

[4 pts] b. Sketch the asymptotic Bode plot for the lag network $D(s)$ alone on the plot below:

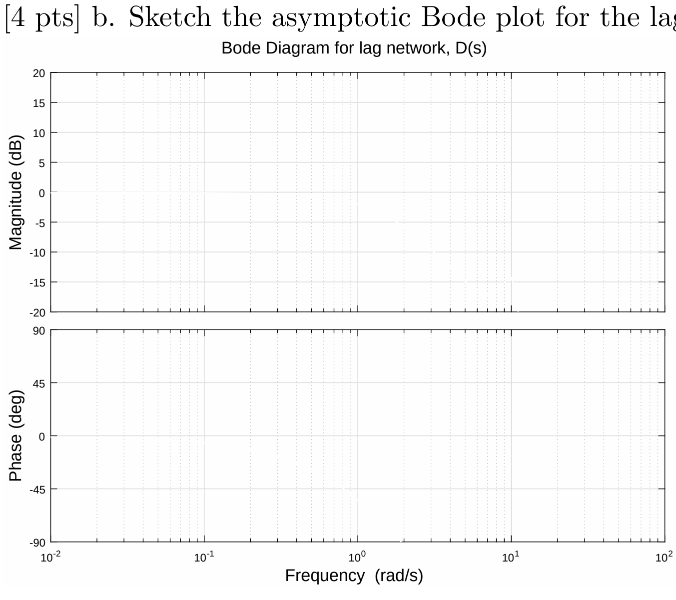


Fig. 3.2

[4 pts] c. Sketch the asymptotic Bode plot for the combined lag network and plant $D(s)G(s)$ on the plot (Fig. 3.1) at top of page.

[2 pts] d. Mark the phase margin and phase margin frequency on the plot of $D(s)G(s)$ (Fig. 3.1).

Problem 3 (18 pts)

[2 pt] a. Given the homogeneous linear differential equation $\dot{\mathbf{x}} = A\mathbf{x}$ with initial condition $\mathbf{x}(0) = \mathbf{x}_o$. Show that the solution $\mathbf{x}(t) = e^{At}\mathbf{x}_o$ satisfies both conditions.

[2 pt] b. Show that e^{At} must equal $\mathcal{L}^{-1}[sI - A]^{-1}$. (Hint: see part a. above.)

[2 pts] c. Given $\bar{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, find $e^{\bar{A}t}$

$$e^{\bar{A}t} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

[4 pts] d. Given $\bar{A}, A, P$ such that $\bar{A} = P^{-1}AP$ is diagonal, and given $e^{\bar{A}t}$. Also given the state vector $x = P\bar{x}$. Show how to find e^{At} given $\bar{A}, A, P, e^{\bar{A}t}$, starting from $\dot{\bar{x}} = \bar{A}\bar{x}$. (Leave in general form.)

$e^{At} =$ ______________________________

Problem 3, cont.

Given the two LTI systems

$$\dot{\mathbf{x}}(t) = A\mathbf{x} + Bu = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u, \quad y = C\mathbf{x} = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{\mathbf{z}}(t) = A_z\mathbf{z} + B_z u = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u \ , \quad y = C_z\mathbf{z} = [0 \quad 1] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

[4 pts] e. Find a transformation P such that $A = P^{-1}A_zP$ is diagonal. (Hint: this could be found using the controllability matrix for each system.)

$$P = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

[4 pts] f. Show that both systems have the same input-output behavior. That is, for the same input $u(t)$, the output $y(t)$ will be identical for both systems. Use P from part e, and also verify B_z and C_z are correct.

Problem 4. (20 pts)

Given the LTI system

$$\dot{\mathbf{x}}(t) = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad\qquad y = C\mathbf{x} = [1 \quad 0]\mathbf{x},$$

[3 pts] a. Find $\mathbf{k} = [k_1 \ k_2]$ such that with state feedback $u = r - \mathbf{kx}$, the closed-loop poles of the system are at λ_1, λ_2.

$k_1 =$ ______ $k_2 =$ ________

[1 pts] b. The initial condition is $\mathbf{x}(0) = [0 \ \ 0]^T$. For $r(t)$ a unit step input, it is required that $x_1(t) < 1 \ \forall t$, that is over shoot is not allowed.

What is range of λ_1, λ_2 to avoid over shoot?

[3 pts] c. Assume $\mathbf{k} = [k_1 \ k_2] = [4 \ \ 5]$. Let $e(t) = r(t) - C\mathbf{x}$. For $r(t)$ a unit step input, find the steady state error.

$\lim_{t\to\infty} e(t) =$ ____________________

[3 pts] d. For $\mathbf{k} = [k_1 \ k_2] = [4 \ \ 5]$, with $u = r - \mathbf{kx}$, find $\frac{Y(s)}{R(s)}$. (Express the transfer function as a ratio of polynomials, not as matrix operations.)

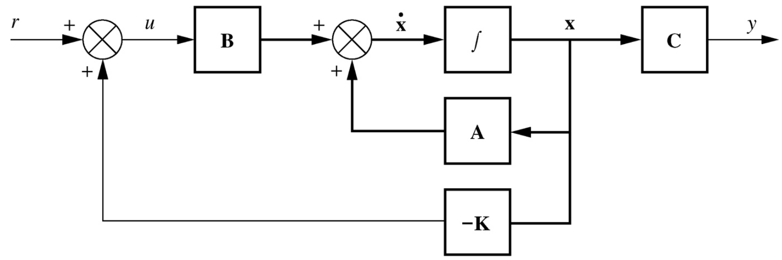
$\frac{Y(s)}{R(s)} =$ __________

Problem 4, cont. (20 pts)

[4 pts] e. Define $e_w(t)$ to be the error between an input $w(t)$ and output $y(t)$. That is, $e_w(t) = w(t) - y(t)$. We desire to find an input $r(w, y)$ to the state feedback system shown below in part f such that $\lim_{t\to\infty} e_w(t) = 0$ for a step input $w(t)$ of any amplitude.

$r(w, y) =$ ____________

[2 pts] f. Using the controller from part e, expand the block diagram below to include the controller and input w.



[4 pts] g. Assume the overall control system is stable, and refer to the expanded block diagram above. Describe in words why $\lim_{t\to\infty} e_w(t) = 0$ for a step input $w(t)$.

Problem 5. 16 pts

Given the following system model:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \;=\; \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k_2 & 1 \\ 0 & -k_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad y = C\mathbf{x} = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[2 pts] a. Determine if the system A, B, C is controllable, and restrictions if any on k_1, k_2 for controllability.

[2 pts] b. Determine if the system A, B, C is observable, and restrictions if any on k_1, k_2 for observability.

[2 pts] c. Provide state equations for an observer which takes as inputs $u(t), y(t)$, and provides an estimate of the state $\hat{\mathbf{x}}(t)$.

[6 pts] d. Given $k_1 = 1, k_2 = 4$, find observer gain L such that the observer has closed loop poles at $s_1 = -10, s_2 = -10$.

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

Problem 5, cont.

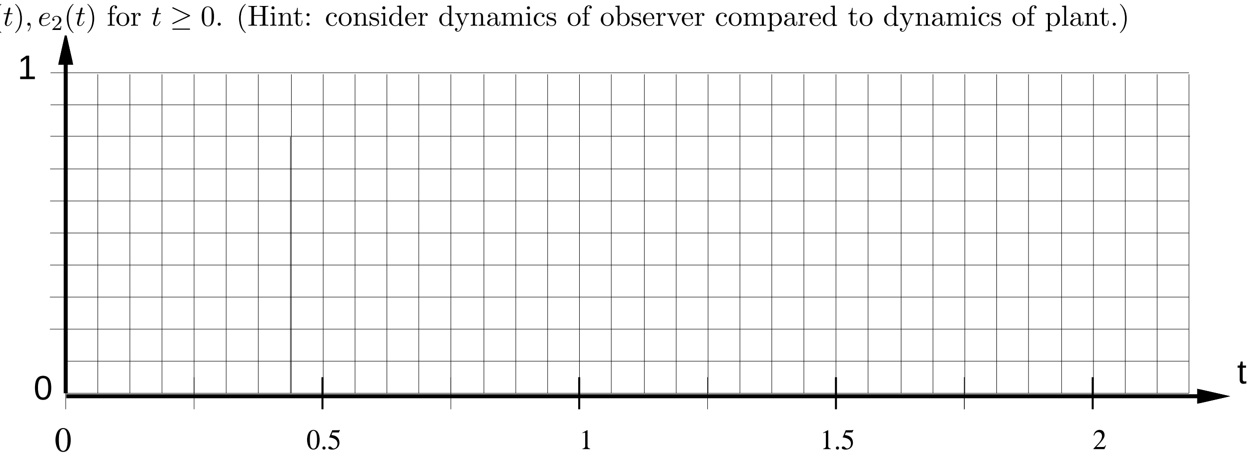
[2 pts] e. Let the error between the estimated state and the true state be given by $\mathbf{e}(t) = \mathbf{x} - \hat{\mathbf{x}}$. Find the dynamics of the error in terms of A, B, C, L.

$\dot{\mathbf{e}} =$ ____________

[2 pts] f. Given initial conditions

$$\mathbf{x} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and observer has closed loop poles at $s_1 = -10, s_2 = -10$, and $k_1 = 1, k_2 = 4$. Sketch approximately $e_1(t), e_2(t)$ for $t \geq 0$. (Hint: consider dynamics of observer compared to dynamics of plant.)



Problem 6 (15 pts)

[3 pts] a. Given the discrete time system below, find $\mathbf{X}(z)$ the z-transform of $\mathbf{x}(k)$, where $u(k) = (\frac{1}{2})^k$ for $k \geq 0$. (Assume $\mathbf{x}(0) = \mathbf{0}$.)

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$\mathbf{X}(z) = \begin{bmatrix} \quad\quad\quad \end{bmatrix}$

[2 pts] b. Given

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

Determine the response of the system to $u(k)$ a unit step input. (Assume $\mathbf{x}(0) = \mathbf{0}$.)

$\mathbf{x}(k) = \begin{bmatrix} \quad\quad\quad \end{bmatrix}$

[4 pts] c. Given

$$X(z) = \frac{z^2}{(z - \frac{1}{2})(z-1)}$$

find $x(k)$ for $k \geq 0$.

$x(k) =$ ____________________

Problem 6, cont.

[2 pts] d. Given

$$X(z) = \frac{z}{(z - \frac{1}{2})(z - 1)(z - \frac{2}{5})}$$

find $\lim_{k \to \infty} x(k) =$ ____________________

[4 pts] e. Given a mass m, and input force f, $\ddot{x} = f/m$. Let the state x_1 be the position and x_2 velocity of the mass. The continuous time state equations for the system are :

$$\dot{\mathbf{x}} = A\mathbf{x} + Bf \;\; = \;\; \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t),$$

Find the discrete time equivalent system using zero-order hold for input force $f(t)$ and sampling period T: $\mathbf{x}((k+1)T) = G\mathbf{x}(kT) + Hf(kT)$.

$$G = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \qquad\qquad H = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

Blank page for scratch work.