Chemical Engineering 150A
Midterm Exam – 2 – Solutions
Wednesday, April 5, 2017
7:10 pm – 8:00 pm

The exam is 100 points total.

Name: ______________________________ (in Uppercase)

Student ID: __________________________

You are allowed one 8.5”×11” sheet of paper with your notes on both sides.

The exam should have 14 pages (front and back) including the cover page.

Instructions:

1) Do your calculations in the space provided for the corresponding part. Any work done outside of specified area (including scratch sheet) will not be graded.
3) Please sign below saying that you agree to the UC Berkeley honor code.
4) The exam contains two problems.
5) Use the last two pages as scratch sheet if you would like to.
6) Exam should be turned in SHARP 8:00 PM.
7) Navier-Stokes, continuity, and Newtonian viscosity equations are provided starting on page 12.

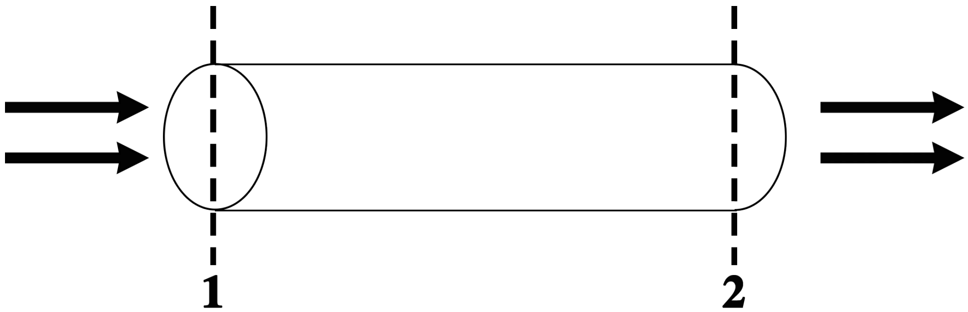
Honor Code:

As a member of the UC Berkeley community, I act with honesty and integrity.

Signature: _

Problem 1: (50 Points)

In a part of a chemical process an ideal gas of molecular mass M must be piped through a straight, horizontal length of pipe. The pipe has an inside diameter of D. The nitrogen flow can be considered turbulent, isothermal, and at steady state at temperature T. The inlet pressure is P_1, and the outlet pressure is P_2. Nitrogen is supplied at a mass flow rate of $\dot{m}$ and you can assume a plug flow profile, and that density and viscosity are constant across the cross section of the pipe.



You may find the energy balance equation reconstituted in terms of Helmholtz free energy, f, to be useful, along with the following thermodynamic identity:

$$\frac{\partial}{\partial t}\iiint_{CV}\left(f+\frac{1}{2}\underline{v}\cdot\underline{v}+gh\right)\rho dV=-\iint_{CS}\left(f+\frac{1}{2}\underline{v}\cdot\underline{v}+gh+\frac{P}{\rho}\right)\rho\underline{v}\cdot\underline{n}dA+E_V+\dot{W}_S$$

$$df=-sdT-Pd\,\tilde{v}$$

where f is the Helmholtz free energy per unit mass, $\tilde{v}$ is the specific volume ($\tilde{v}=\frac{1}{\rho}$), and s is the entropy per unit mass. Also, f is a function of density (or specific volume) and temperature. Note the difference between the meaning of the symbols for specific volume $\tilde{v}$ and the velocity $\underline{v}$.

MAKE ALL THE ASSUMPTIONS THAT YOU FEEL ARE PHYSICAL. Note that this problem has parts (a) and (b).

a) What is the change in Helmholtz free energy (f_2-f_1) in the pipe? Your answer may only include R, T, P_2, P_1, M, D, and $\dot{m}$, where R is the universal gas constant.

Use the thermodynamic relation to determine $f_2 - f_1$:

$$df = -sdT - Pd\tilde{v}$$

Isothermal, so $df = -Pd\tilde{v} = -Pd\left(\frac{1}{\rho}\right) = -P\left(-\frac{1}{\rho^2}\right)d\rho = \frac{P}{\rho^2}d\rho$

Leverage the ideal gas equation of state.

$$PV = nRT = \frac{mRT}{M}$$

$$P = \frac{\rho RT}{M}$$

$$\rho = \frac{PM}{RT}$$

Combine.

$$df = \frac{\rho RT}{M} * \frac{1}{\rho^2}d\rho = \frac{RT}{M\rho}d\rho$$

Integrate.

$$f_2 - f_1 = \frac{RT}{M}\ln\frac{\rho_2}{\rho_1}$$

Rewrite ρ in terms of P using the ideal gas law.

$$\boldsymbol{f_2 - f_1 = \frac{RT}{M}\ln\frac{P_2}{P_1}}$$

b) Calculate the **total** viscous loss E_V in the pipe. Your final answer may include only the following variables: the gas constant R, T, P_2, P_1, M, D, and $\dot{m}$. If you were not able to solve part (a), you may leave the variable f in your equation for full credit.

Steady flow, no shaft work, no change in height.

$$0 = -\iint_{CS}\left(f + \frac{1}{2}\underline{v}^2 - \frac{P}{\rho}\right)\rho\underline{v}\cdot\underline{n}dA + E_V$$

Evaluate at control surfaces.

$$E_V = \left(f_2 + \frac{1}{2}\langle v_2\rangle^2 + \frac{P_2}{\rho_2}\right)(\rho_2\langle v_2\rangle A) - \left(f_1 + \frac{1}{2}\langle v_1\rangle^2 + \frac{P_1}{\rho_1}\right)(\rho_1\langle v_1\rangle A)$$

Mass balance.

$$(\rho_1\langle v_1\rangle A) = (\rho_2\langle v_2\rangle A) = \dot{m}$$

Substitute.

$$E_V = \left(\frac{1}{2}\langle v_2\rangle^2 + \frac{P_2}{\rho_2}\right)\dot{m} - \left(\frac{1}{2}\langle v_1\rangle^2 + \frac{P_1}{\rho_1}\right)\dot{m} - \dot{m}(f_2 - f_1)$$

Use the ideal gas law to rewrite P/ρ terms in terms of only P.

$$\rho = \frac{M}{RT}P$$

Substitute.

$$E_V = \left(\frac{1}{2}\langle v_2\rangle^2 + \frac{RT}{M}\frac{P_2}{P_2}\right)\dot{m} - \left(\frac{1}{2}\langle v_1\rangle^2 + \frac{RT}{M}\frac{P_1}{P_1}\right)\dot{m} - \dot{m}(f_2 - f_1)$$

Cancel identical terms.

$$E_V = \left(\frac{1}{2}\langle v_2\rangle^2\right)\dot{m} - \left(\frac{1}{2}\langle v_1\rangle^2\right)\dot{m} - \dot{m}(f_2 - f_1)$$

Take advantage of mass balance.

$$\langle v\rangle = \frac{\dot{m}}{\rho A} = \frac{m\dot{R}T}{MAP}$$

Substitute.

$$E_V = \left(\frac{1}{2}\frac{\dot{m}^2\dot{R}^2T^2}{M^2A^2P_1^2}\right)\dot{m} - \left(\frac{1}{2}\frac{\dot{m}^2R^2T^2}{M^2A^2P_2^2}\right)\dot{m} - \dot{m}(f_2 - f_1)$$

Simplify.

$$E_V = \frac{\dot{m}RT}{M}\left[\frac{1}{2}\frac{\dot{m}^2RT}{MA^2P_1^2} - \frac{1}{2}\frac{\dot{m}^2RT}{MA^2P_2^2}\right] - \dot{m}(f_2 - f_1)$$

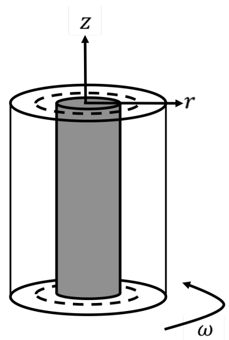
$$E_V = \frac{\dot{m}RT}{M}\left[8\frac{\dot{m}^2RT}{\pi^2MD^4P_1^2} - 8\frac{\dot{m}^2RT}{\pi^2MD^4P_2^2}\right] - \dot{m}(f_2 - f_1)$$

Substitute in for $f_2 - f_1$ (optional)

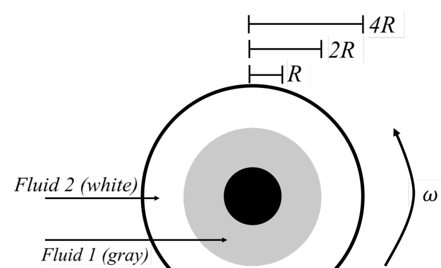
$$\boldsymbol{E_V = \frac{\dot{m}RT}{M}\left[8\frac{\dot{m}^2RT}{\pi^2MD^4P_1^2} - 8\frac{\dot{m}^2RT}{\pi^2MD^4P_2^2} - \ln\frac{P_2}{P_1}\right]}$$

Problem 2. (50 points)

Two immiscible and incompressible, Newtonian fluids fill the annular region between two infinitely long cylinders. The inner cylinder is stationary while the outer cylinder rotates with an angular velocity ω. The inner cylinder has radius R and the outer cylinder has radius $4R$. Fluid 1 is in the region between $r = R$ and $r = 2R$; and Fluid 2 is the region between $r = 2R$ and $r = 4R$. The cylinder is oriented vertically with gravity acting solely in the *z*-direction. Fluid 1 has a viscosity of 12μ and Fluid 2 has a viscosity of μ. Consider the densities of the two fluids to be the same as ρ. The figures below provide a schematic representation of the problem from a side view of the cylinder and a top view. These figures are not necessarily to scale.



Side View



Top View

Assuming that the flow is well developed and at steady-state, what is the velocity profile for fluid flow in between the two cylinders as a function of radius?

NOTE:
1. Assume that the velocity profile is not a function of height *z*.
2. Make all the assumptions that seem physical to you.
3. Make sure to write the relevant equations and the appropriate boundary conditions to get partial credit.
4. Your final answer may contain the following constants only – gravity g, viscosities μ, radius R, density ρ, angular velocity ω. Simplify your answer as much as possible.

Steady state, incompressible Newtonian fluid.

Since the outer cylinder is rotating in θ direction, we can assume $v_r = 0, v_z = 0$.

Also, from symmetry we can assume $v_\theta \neq v_\theta\ (\theta)$

Since the fluid flow is fully developed in z direction, $v_\theta \neq v_\theta\ (z)$

Thus, from kinematics $v_\theta = v_\theta\ (r)$

Simplifying the θ component for Navier Stokes equation:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\rho g_\theta+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)=0$$

Since the two fluids have different viscosities, the velocity profile needs to be solved separately for each region.

For fluid 1 in the region $R < r < 2R$:

$$\mu_1\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_{\theta,1})\right)=0$$

For fluid 2 in the region $2R < r < 4R$:

$$\mu_2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_{\theta,2})\right)=0$$

Integrating twice:

$$v_{\theta,1}(r)=\frac{1}{2}c_1 r+\frac{c_2}{r} \qquad (1)$$

$$v_{\theta,2}(r)=\frac{1}{2}c_3 r+\frac{c_4}{r} \qquad (2)$$

Boundary conditions (BC):
1) No slip: $v_{\theta,1}(R) = 0$
2) No slip: $v_{\theta,2}(4R) = 4wR$
3) Continuity of velocity at the fluid interface: $v_{\theta,1}(2R) = v_{\theta,2}(2R)$
4) Continuity of shear stress at the fluid interface: $\tau_{r\theta,1}(2R) = \tau_{r\theta,2}(2R)$

Using BC 1 in eq (1), we get

$$c_1=-\frac{c_2}{R^2}$$

Using BC 2 in eq (2), we get

$$c_3=\omega-\frac{c_4}{16R^2}$$

$$\tau_{r\theta,1} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_{\theta,1}}{r}\right)\right] = -\mu_1\frac{c_2}{r^2} \quad (3)$$

$$\tau_{r\theta,2} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_{\theta,2}}{r}\right)\right] = -\mu_2\frac{c_4}{r^2} \quad (4)$$

Using BC 4 and eq (3) and (4), we get

$$\mu_1 c_2 = \mu_2 c_4$$

Thus,

$$c_4 = 12c_2$$

Using BC 2 and eq (1 and (2), we get

$$c_1 R + \frac{c_2}{4R} = c_3 R + \frac{c_4}{4R}$$

Substituting for c_1, c_3 and c_2, we get

$$c_4 = -4\omega R^2$$

$$c_2 = -\frac{\omega R^2}{3}$$

$$c_1 = \frac{\omega}{3}$$

$$c_3 = \frac{5}{4}\,\omega$$

Thus, we get the following velocity profile for the two fluids:

For fluid 1 in the region $R < r < 2R$:

$$\boldsymbol{v_{\theta,1}(r) = \frac{1}{3}\omega r - \frac{\omega}{3r}\, R^2}$$

For fluid 2 in the region $2R < r < 4R$:

$$\boldsymbol{v_{\theta,2}(r) = \frac{5}{4}\omega r - \frac{4\omega}{r}\, R^2}$$