ME 109 Final Solutions

Problem 1

$$q = A\sigma\left(T_{eq}^4 - T_{sur}^4\right) \Rightarrow$$

$$1000 = \pi \cdot \frac{0.2^2}{4}\, 5.67\cdot 10^{-8}\left(T_{eq}^4 - 500^4\right) \Rightarrow$$

$$T_{eq}^4 = \frac{1000}{\pi \cdot \frac{0.2^2}{4} \cdot 5.67\cdot 10^{-8}} + 500^4 \Rightarrow T_{eq} = 888K$$

$$q_{0.3\rightarrow 1.5} = \sigma T_{eq}^4 \left[F_{0\rightarrow 6\cdot T_{eq}} - F_{0\rightarrow 0.3\cdot T_{eq}} \right]$$

$$f = F_{0\rightarrow 6\cdot T_{eq}} - F_{0\rightarrow 2\cdot T_{eq}}$$

$$f = F_{0\rightarrow 5328\ \mu m K} - F_{0\rightarrow 1776\ \mu m K}$$

$$\downarrow$$

$$\sim 0.67 \quad - \quad 0.04$$

$$\boxed{f \simeq 0.63}$$

Problem 2

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$\nu = \frac{\mu}{\rho} = \frac{420\cdot10^{-6}}{980} = 4.28\cdot10^{-7}\ m^2/s$$

$$Gr_L = \frac{9.81\cdot506\cdot10^{-6}\cdot(340-280)\cdot0.2^3}{(4.28\cdot10^{-7})^2} \Rightarrow$$

$$Gr_L = 1.3\cdot10^8 \quad \text{laminar.}$$

$$Ra = 3.45\,10^8$$

$$\overline{Nu}_L = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} g(Pr)$$

$$g(Pr) = \frac{0.75\,Pr^{1/2}}{(0.609 + 1.221\,Pr^{1/2} + 1.238\,Pr)^{1/4}}$$

$$= \frac{0.75\cdot2.66^{1/2}}{(0.609 + 1.221\cdot2.66^{1/2} + 1.238\cdot2.66)^{1/4}}$$

$$= 1.91$$

$$\overline{Nu}_L = \frac{4}{3}\left(\frac{1.3\cdot10^8}{4}\right)^{0.25}\cdot 1.91 \Rightarrow$$

$$\overline{Nu}_L = 192.3$$

$$\overline{Nu}_L = \frac{\bar{h} L}{k} \Rightarrow \bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{192.3 \cdot 0.66}{0.2} \Rightarrow$$

$$\bar{h} = 634.5 \ W/m^2K$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo) \qquad L_c = \frac{V}{A} = \frac{L^2 d}{2L^2} = \frac{d}{2}$$

$$Bi = \frac{\bar{h} \cdot d/2}{k} = \frac{634.5 \ 0.5 \cdot 10^{-3}}{40} \Rightarrow$$

$$Bi = 7.93 \cdot 10^{-3} \quad (\text{lumped ok})$$

$$\frac{320 - 280}{360 - 280} = \exp(-7.93 \cdot 10^{-3} \ Fo)$$

$$-\ln\left(\frac{40}{80}\right) \frac{1}{7.93 \cdot 10^{-3}} = Fo \Rightarrow$$

$$Fo = 87.4$$

$$\alpha = \frac{k}{\rho c_p} = \frac{401}{8933 \cdot 385} \Rightarrow$$

$$\alpha = 1.1 \cdot 10^{-5} \ m^2/s$$

$$\frac{\alpha t}{L_c^2} = Fo \Rightarrow t = \frac{L_c^2 \ Fo}{\alpha}$$

$$t = \frac{(0.5 \cdot 10^{-3})^2 \cdot 87.4}{1.1 \cdot 10^{-5}} \Rightarrow$$

$$\boxed{t = 1.98 \ s}$$

Problem 3

(a) $\dot{m} = \rho u_m A_c = 988 \cdot 10 \cdot \frac{\pi}{4} (0.05)^2 = 19.1 \text{ Kg/s}$

$$Re_D = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4 \cdot 19.1}{\pi \, 0.05 \cdot 528 \cdot 10^{-6}} = 9.21 \cdot 10^5$$

turbulent!

Dittus-Boelter

$$Nu_D = 0.023 \, Re_D^{4/5} \, Pr^{0.3}$$

$$= 0.023 \cdot (9.21 \cdot 10^5)^{4/5} \cdot 3.42^{0.3}$$

$$Nu_D = 1964$$

$$Nu_D = \frac{h_{int} D_i}{K} \Rightarrow h_{int} = \frac{Nu_D \cdot K}{D_i} = \frac{1964 \cdot 0.645}{0.05} \Rightarrow$$

$$h_{int} = 25336 \text{ W/m}^2\text{K}$$

(b) External

$$Re_D = \frac{VP}{\nu} = \frac{20 \cdot 0.10}{15.89 \cdot 10^{-6}} = 1.26 \cdot 10^5 = 126000$$

$$\overline{Nu_D} = C \, Re_D^m \, Pr^{1/3}$$

$C = 0.027 \quad m = 0.805$

$$\overline{Nu_D} = 0.027 \cdot (126000)^{0.805} \cdot 0.707^{1/3}$$

$$\overline{Nu_D} = 306.9$$

$$\frac{\bar{h} D_o}{k} = 4306.9 \Rightarrow$$

$$\bar{h} = \frac{4306.9 \cdot 26.3 \cdot 10^{-3}}{0.1} \Rightarrow$$

$$\bar{h} = 80.7 \ W/m^2K$$

(c)

$$\frac{1}{UA_s} = \frac{1}{h_{int} \pi D_i L} + \frac{\ln(D_o/D_i)}{2\pi L K} + \frac{1}{h_{out} \pi D_o L}$$

$$= \frac{1}{2538 \cdot \pi \cdot 0.05 \cdot 40} + \frac{\ln(10/5)}{2\pi \cdot 40 \cdot 10} + \frac{1}{80 \cdot \pi \cdot 0.1 \cdot 40}$$

$$\frac{1}{UA_s} = 6.27 \cdot 10^{-6} + 2.75 \cdot 10^{-4} + 9.95 \cdot 10^{-4}$$

$$UA_s = (1.33 \cdot 10^{-3})^{-1} = 7.5 \cdot 10^2$$

$$\frac{T_\infty - T_{mo}}{T_\infty - T_{mi}} = \exp\left(-\frac{UA_s}{\dot{m} C_p}\right)$$

$$\frac{300 - T_{mo}}{300 - 350} = \exp\left(-\frac{750}{19.1 \cdot 4.18 \cdot 10^3}\right)$$

$$\frac{300 - T_{mi}}{(-50)} = 0.99 \Rightarrow$$

$$300 - T_{mo} = 0.99 \ (-50) = -49.5$$

$$T_{mo} = 300 + 49.5 \Rightarrow \boxed{T_{mo} = 349.5 \ K}$$

(d) $q = \dot{m} C_p (T_{mi} - T_{mo})$

$= 19.1 \cdot 4.18 \cdot 10^3 \cdot 0.05 \Rightarrow$

$q = 3.99 \cdot 10^3 \text{ W}$

Problem 4

$$q_s \cdot \cos\theta = \alpha_s = \varepsilon \sigma T^4$$

$$\alpha_s = 0.9 \cdot F_{0 \to 2.5800} + 0.1 \cdot (1 - F_{0 \to 2.5800})$$

$$= 0.9 \cdot F_{0 \to 11,600} + 0.1 \left(1 - F_{0 \to 11600}\right)$$

$$0.9 \quad 0.94 + 0.1 (1 - 0.94)$$

$$\alpha_s = 0.85$$

$$\varepsilon = 0.9 \, F_{0 \to \{2.500} + 0.1 \left(1 - F_{0 \to 2.500}\right) \Rightarrow$$

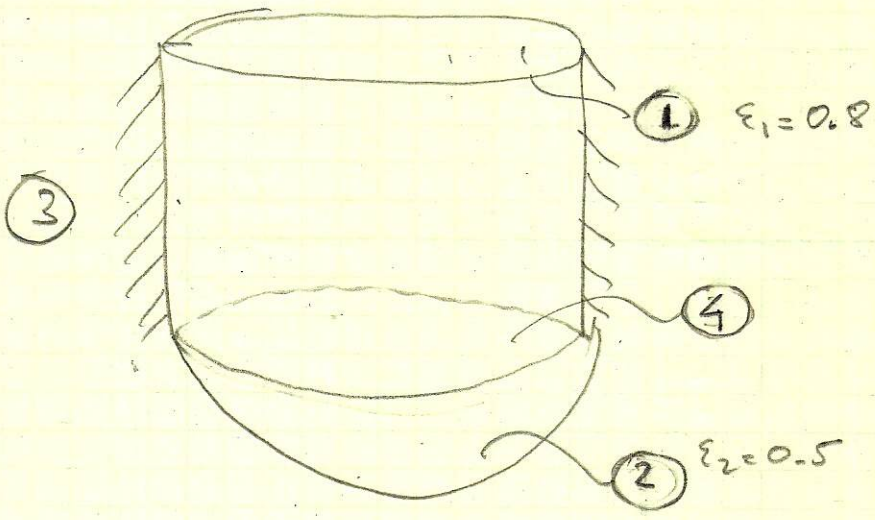
$$\varepsilon = 0.1$$

$$\cos\theta = \frac{0.1 \cdot 5.67 \cdot 10^{-8} \cdot 500^4}{1353 \cdot 0.85} \Rightarrow$$

$$\cos\theta = 0.308 \Rightarrow$$

$$\theta = 72°$$

Problem 5



$F_{1-2} = F_{1-4}$

$R_1 = \frac{r_1}{L} = \frac{0.75}{0.8} = 0.937$

$R_2 = \frac{r_2}{L} = 0.937$

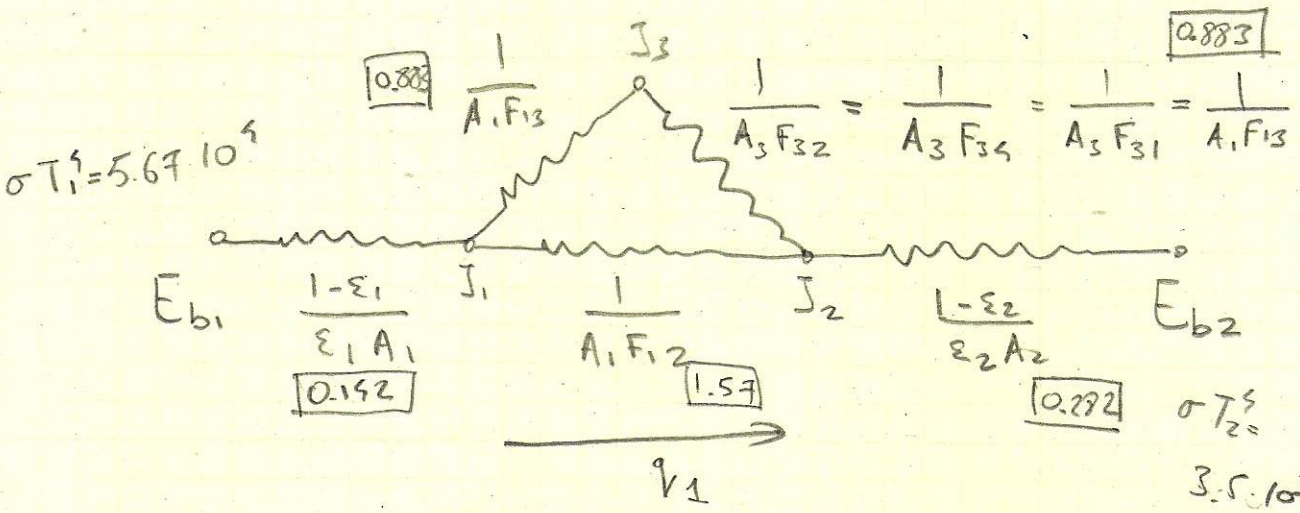
$S = 1 + \frac{1 + R_j^2}{R_i^2} = 1 + \frac{1 + 0.937^2}{0.937^2} = 3.14$

$F_{12} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2 \right]^{1/2} \right\}$

$= \frac{1}{2}\left\{ 3.14 - \left[3.14^2 - 4\ 1 \right]^{1/2} \right\} = 0.359$

$F_{13} + F_{14} = 1 \Rightarrow F_{13} = 1 - 0.359 = 0.641$

$F_{31} + F_{34} + F_{33} = 1$



$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.2}{0.8\cdot\frac{\pi\cdot1.5^2}{4}} = 0.142$$

$$\frac{1}{A_1F_{12}} = \frac{1}{\frac{\pi\cdot1.5^2}{4}\cdot 0.359} = 1.57 \qquad \frac{1}{A_1F_{13}} = \frac{1}{\frac{\pi\cdot1.5^2}{4}\cdot 0.641} = 0.883$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{0.5}{0.5\cdot 2\pi\cdot 0.75^2} = 0.282$$

$$q_1 = \frac{5.67\cdot10^4 - 3.5\cdot10^3}{0.142 + \frac{1}{\frac{1}{1.57} + \frac{1}{0.883+0.883}} + 0.282} = 42401\ W$$

$$E_{b1} - J_1 = q_1 \frac{1-\varepsilon_1}{\varepsilon_1 A_1} \Rightarrow$$

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 5.67 \cdot 10^4 - 42401 \cdot 0.142 \Rightarrow$$

$$J_1 = 50679 \ W/m^2$$

$$J_2 = E_{b2} + q_1 \frac{1-\varepsilon_2}{\varepsilon_2 A_2} \Rightarrow$$

$$J_2 = 3.5 \cdot 10^3 + 42401 \cdot 0.282 \Rightarrow$$

$$J_2 = 15457 \ W/m^2$$

$$q_1 = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} \Rightarrow$$

$$42401 = \frac{50679 - 15457}{1.57} + \frac{50679 - J_3}{0.883} \Rightarrow$$

$$-19967 \cdot 0.883 + 50679 = J_3 \Rightarrow J_3 = 33048 \frac{W}{m^2}$$

$$J_3 = \sigma T_3^4 \Rightarrow$$

$$T_3 = \left(\frac{J_3}{\sigma}\right)^{0.25} = \left(\frac{33048}{5.67 \cdot 10^{-8}}\right)^{0.25} \Rightarrow$$

$$T_3 = 873 \ K$$