Solutions

Final Examination

May 12, 2007

12:30 PM to 3:30 PM

@ 180 Tan Hall

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ID: ____________________

Problem 1 __________/20

Problem 2 __________/20

Problem 3 __________/20

Problem 4 __________/20

Problem 5 __________/20

Problem 6 __________/20

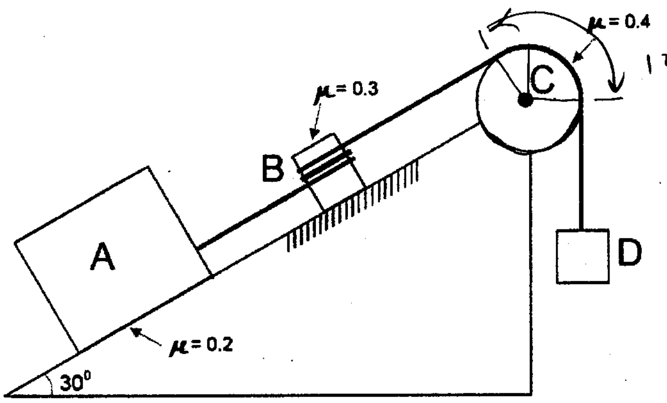
TOTAL: __________/120

1. A block weighing 3000 lbs is resting on a slope of 30° inclination and is held in position by a weightless rope as shown. The rope wraps 2 turns around a cylindrical bollard of 1′ in diameter, which is perpendicular to the slope, and then the rope goes over a **frozen** cylindrical pulley of 2′ in diameter. To prevent the block from sliding down, a counter weight is hung at the end, D, of the rope. The static friction coefficients between block and slope, rope and bollard, rope and frozen pulley are 0.2, 0.3 and 0.4, respectively. The rope is parallel to the slope. Calculate the **lightest** counter weight required to prevent the block from sliding down the slope.

   (1) Rope tension between A and B = 980.38 lbs

   (2) Rope tension between B and C = 22.60 lbs

   (3) Minimum weight of the counter weight = 9.78 lbs

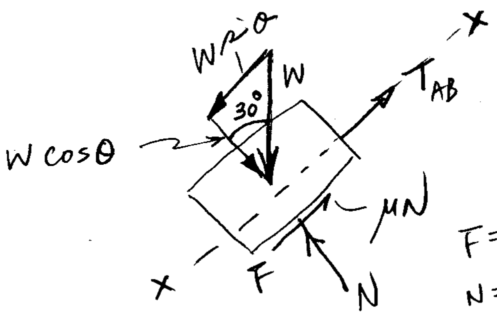


$120° = \frac{2}{3}\pi$

$T_{AB} = T_{BC}\, e^{\mu(4\pi)}$

$T_{BC} = T_{AB}\, e^{-\mu(4\pi)}$ $\quad \mu = 0.3$

$= 22.60$ lbs



$F = \mu N$

$N = W\cos\theta$

$T_{BC} = T_{CD}\, e^{\mu\left(\frac{2\pi}{3}\right)}$

$T_{CD} = T_{BC}\, e^{-\mu\left(\frac{2\pi}{3}\right)}$

$T_{CD} = 9.78$ lbs

$\sum F_x = 0$

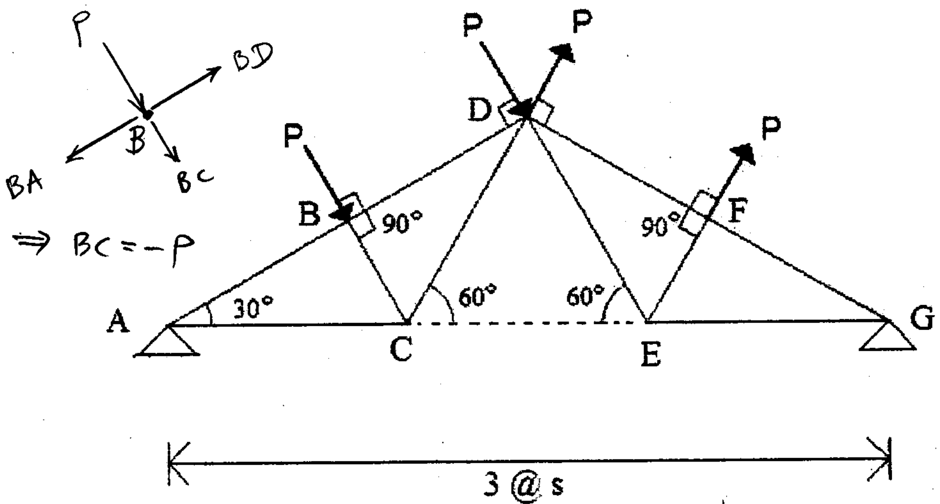
$-W\sin 30° + (W\cos 30°)\mu + T_{AB} = 0$ $\quad (W = 3000,\ \mu = 0.2)$

$T_{AB} = 3000\sin 30° - (3000\cos 30°) \times 0.2$

$= 980.38$ lbs.

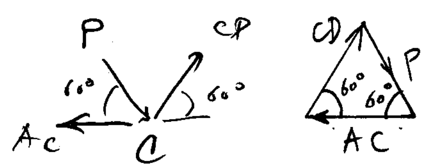
2. A roof truss is subjected to a loading system as shown. The weight of truss members is neglected. **Calculate reaction force components at Supports A and G and forces in ALL members.** Put the results in the table. You must indicate the directions of reaction components by arrows (←↑↓→) and indicate a tensile force by + and compressive force by –.

A_X	$P \leftarrow$	AB	0	CD	$+P$	GF	0
A_Y	0	AC	$+P$	ED	$-P$	FD	0
G_X	$P \leftarrow$	BC	$-P$	EF	$+P$	--	(N/A)
G_Y	0	BD	0	EG	$-P$	--	(N/A)



$\Rightarrow BC = -P$

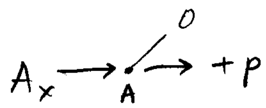
joint C



$CD = +P$

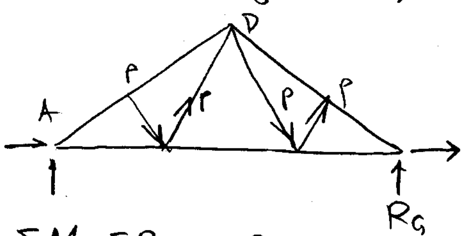
$CA = +P$

joint A



$A_x = -P \leftarrow$

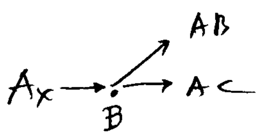
slide roof loads to bottom chord



$\Sigma M_A = 0 \rightarrow G_y = 0$

similarly $A_y = 0$

joint A



$\Rightarrow AB = 0 \Rightarrow BD = 0$

joint B $\quad BC = -P$

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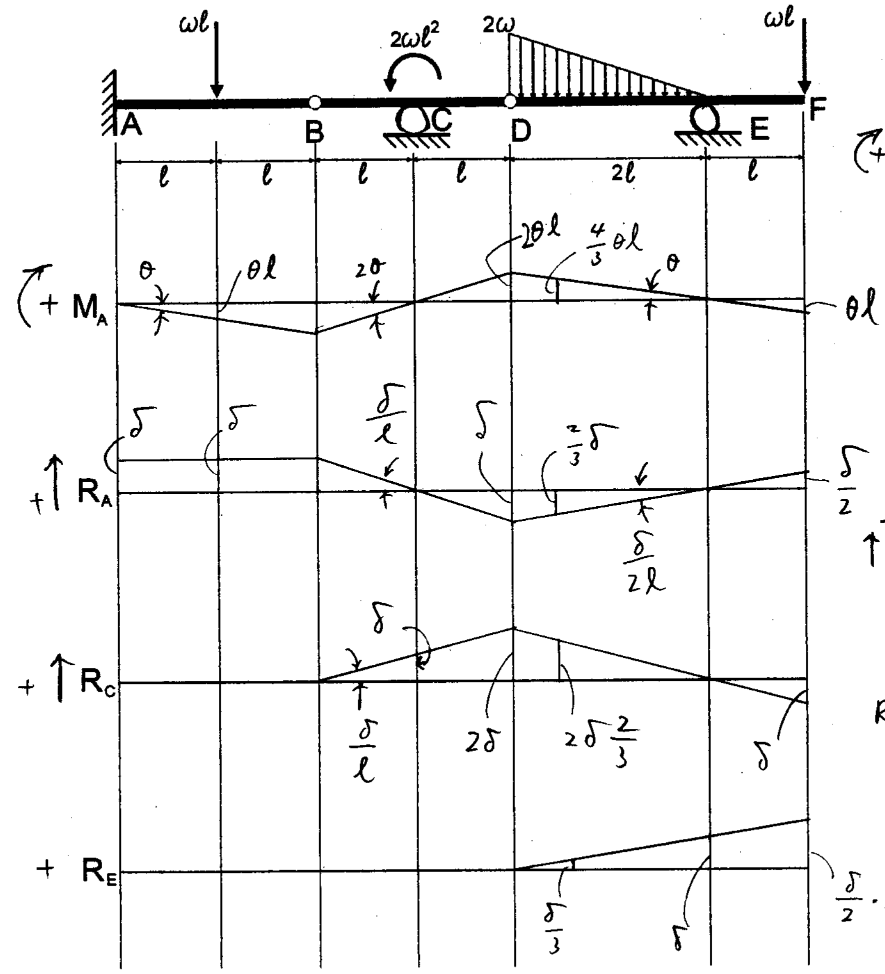
3. A weightless beam consisting of three segments connected by hinges at B and D is subjected to loadings as shown. Use the method of virtual work to find reaction forces at A, C, E and reaction moment at A. Draw the virtual displacements in the figure. Use arrows to indicate directions of reaction forces and clockwise or counterclockwise 'circular arrow' for the reaction moment. **(Note: NO points given if the method of virtual work is not used.)**

Reactions: R_A $\frac{13}{6}w\ell \uparrow$ M_A $\circlearrowright \frac{10}{3}w\ell^2$ R_C $\frac{1}{3}w\ell \downarrow$ R_E $\frac{13}{6}w\ell \uparrow$



$\circlearrowright +$ $M_A\theta + w\ell\theta\ell$
$+ 2w\ell^2 2\theta - (2w\ell)\frac{4}{3}\theta\ell$
$+ w\ell\theta\ell$

$M_A = -w\ell^2 - 4w\ell^2 + \frac{8}{3}w\ell^2 - w\ell^2$
$= (\frac{8}{3}-1-4-1)w\ell^2 = -\frac{10}{3}w\ell^2 \circlearrowright$

$\uparrow + \quad R_A\delta - w\ell\delta - 2w\ell^2\frac{2\delta}{\ell} + (2w\ell)\frac{2}{3}\delta$
$- w\ell\frac{\delta}{2}$

$R_A = w\ell + 2w\ell - \frac{4}{3}w\ell + \frac{1}{2}w\ell$
$= (1+2-\frac{4}{3}+\frac{1}{2})w\ell = \frac{13}{6}w\ell \uparrow$

$\frac{\delta}{2}\cdot 3 = 1.5\delta$

$R_C\delta + 2w\ell^2\frac{\delta}{\ell} - 2w\ell\, 2\delta\frac{2}{3} + w\ell\delta = 0$

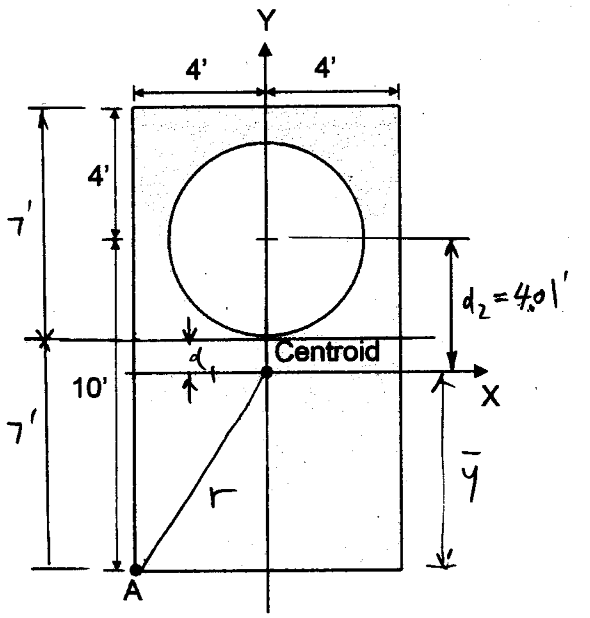
$R_C = -2w\ell + \frac{8}{3}w\ell - w\ell = (\frac{8}{3}-3)w\ell = -\frac{1}{3}w\ell \downarrow$

$R_E\cdot\delta - 2w\ell\cdot\frac{\delta}{3} - w\ell\frac{3}{2}\delta = 0$

$R_E = \frac{2}{3}w\ell + \frac{3}{2}w\ell = \frac{13}{6}w\ell \uparrow$

4. An 8′×14′ rectangular area has one circular hole of 6′ in diameter as shown. Calculate the centroidal moments of inertia I_X and I_Y and the polar moment of inertia J_A about the lower left corner point A of the area.
(Hint: calculate the distance between the centroid and the bottom of the rectangle first.)

Reactions: I_X $1425.31\ ft^4$ I_Y $533.71\ ft^4$ J_A $6302.95\ ft^4$



$$\bar{y} = \left[(14\times 8)\times\left(\tfrac{14}{2}\right) - \pi 3^2 \times 10\right] / A$$

$$= 501.26/83.73$$

$$= 5.99\ ft$$

$$d_1 = 7 - 5.99 = 1.01'$$

$$I_x = \frac{1}{12}\, 8\cdot 14^3 + 14\times 8\times d_1^2 - \frac{\pi 3^4}{4} - (\pi 3^2)(4.01)^2$$

$$= 1829.33 + 114.25 - 63.62 - 454.65$$

$$= \underline{1425.31}$$

$$I_y = \frac{1}{12}\, 14\cdot 8^3 - \frac{\pi 3^4}{4} = 597.33 - 63.62$$

$$= \underline{533.71}$$

$$A = 14\times 8 - \pi 3^2$$

$$= 83.73\ ft^2$$

$$r^2 = 5.99^2 + 4^2 = 51.88$$

$$J = I_x + I_y + A r^2$$

$$= 1425.31 + 533.71 + \underbrace{83.73\,(51.88)}_{4343.92}$$

$$= \underline{6302.95}$$

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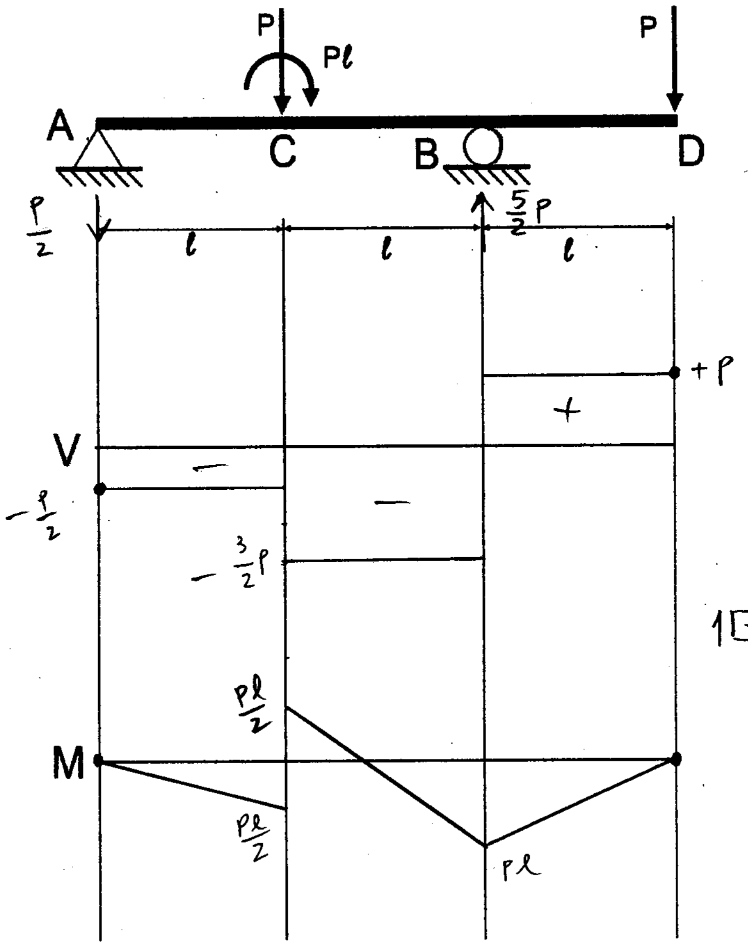
5. A weightless beam is subjected to (1) a vertical load P and a clockwise moment Pl at the middle location C between Supports A and B and a vertical load P at the right end D as shown. Calculate the reactions and draw the shear and moment diagrams. You must indicate the direction of a reaction by an arrow (↑ or ↓).

Ay $\frac{P}{2}$ ↓ By $\frac{5}{2}P$ ↑ Vmax $\frac{3}{2}P$ Mmax Pl



$\Sigma M_A = 0$

$-Pl - Pl + R_B 2l - P3l = 0$

$R_B = 5Pl/2l = \frac{5}{2}P$ ↑

$\Sigma M_B = 0$

$-R_A 2l - Pl + Pl - Pl = 0$

$R_A = -Pl/2l = -\frac{P}{2}$ ↓

↑$\boxed{+}$↓

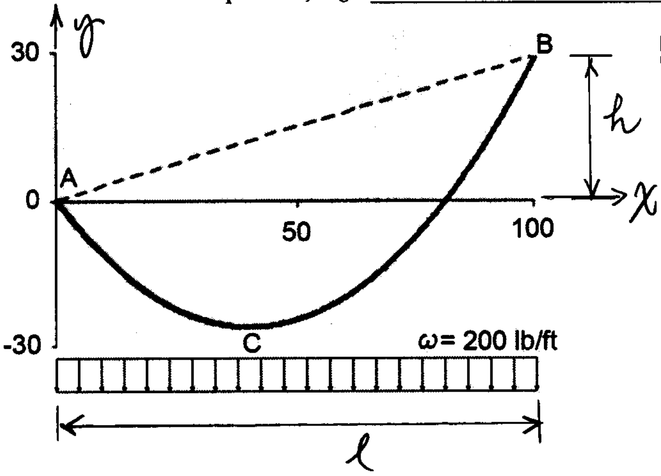
A ↓ $-\frac{P}{2}$

D ↓ $+P$

6. A weightless cable spans between Supports A and B. Support B is 100′ to the right of and 30′ above Support A as shown. The cable is carrying a uniform distributed vertical load of 200 lbs/ft. The cable force at the lowest point C, which is below Support A, was measured to be 6250 lbs. Calculate the maximum cable force and maximum sag (drape) and the position of the lowest point of the cable measured from Support A.

Maximum cable force = 13419 lbs ; Maximum sag = 40′

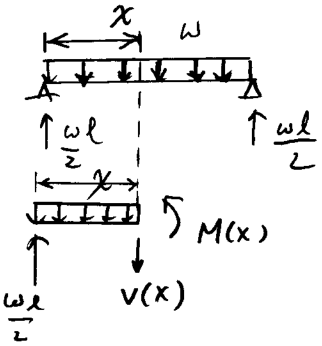
Position of the lowest point C, X_C = 40.625′ ; Y_C = −26.406′



$$y(x) = -\frac{M(x)}{H} + \frac{h}{l}x \quad \text{—— (1)}$$

M(x) is the bending moment of a simply supported beam of the same length l subjected to the same loading.

$H = 6250^{lb}$ ≡ tension @ lowest point

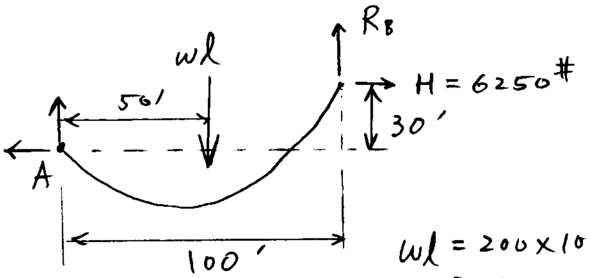


$$M(x) = \frac{1}{2}wlx - wx\frac{x}{2}, \quad V(x) = \frac{1}{2}wl - wx$$

$$= \frac{1}{2}wlx - \frac{1}{2}wx^2$$

max M @ middle $\Rightarrow M_{max} = \frac{1}{8}wl^2$

$$\text{Max. Sag} = \frac{M_{max}}{H} = \frac{\frac{1}{8}200\times100^2}{6250} = 40'$$



$wl = 200\times100$
$= 20000$

FBD

$\Sigma M_A = 0$

$$-20000\times50 - 6250(30) + R_B\times100 = 0$$

$$R_B = 11875^{\#}$$

$$T_{max} = \sqrt{R_B^2 + H^2} = \sqrt{11875^2 + 6250^2}$$

$$= 13419^{lbs}$$

@ lowest point c $y' = 0$

$$y' = -\frac{1}{H}\frac{dM}{dx} + \frac{h}{l} = -\frac{1}{H}V(x) + \frac{h}{l} = 0$$

$$V(x) = \frac{h}{l}H$$

$$V(x) = \frac{1}{2}wl - wx = \frac{h}{l}H$$

$$x_c = \left(\frac{1}{2}wl - \frac{h}{l}H\right)/w$$

$$= \frac{1}{2}l - \frac{h}{l}\frac{H}{w}$$

$$= 50 - \frac{30}{100}\frac{6250}{200}$$

$$= 40.625'$$

use eq. (1) $\Rightarrow y_c = -26.406'$
(you do it!)