EECS C128/ ME C134
Midterm (v. 1.01)
Thurs. Mar. 9, 2017
0810-0930 am

Name: Keys
SID: /

- Closed book. One page formula sheet. No calculators.

- There are 5 problems worth 100 points total.

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Problem	Points	Score
1	22	
2	22	
3	14	
4	27	
5	15	
TOTAL	100	

Tables for reference:

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1}\frac{2}{3} = 33.7°$	$\tan^{-1}\frac{3}{4} = 36.9°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
	$\sqrt{5} \approx 2.24$	$\sqrt{7} \approx 2.65$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 (22 pts)

Key

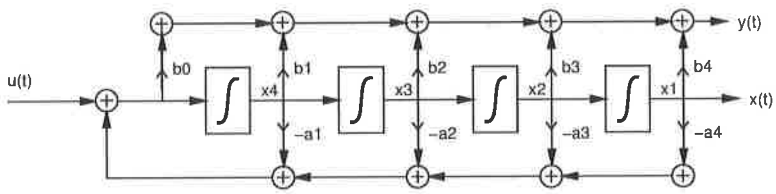
Each part is independent.

[6 pts] a) Consider a single-input single-output system with transfer function:

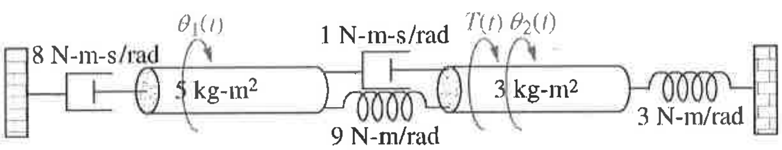
$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 4s + 8}{s^2 + 6s + 10}$$

input $u(t)$ and output $y(t)$. Choose coefficients such that the block diagram below has the same transfer function:

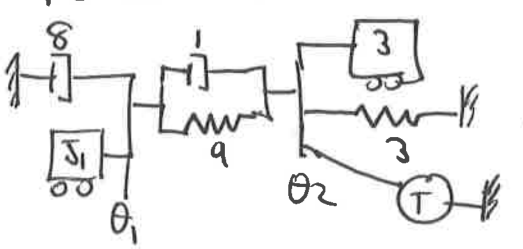
$b_0 = $ 2 $\quad a_1 = $ 6 $\quad b_1 = $ 4 $\quad a_2 = $ 10 $\quad b_2 = $ 8 $\quad a_3 = $ 0 $\quad b_3 = $ 0 $\quad a_4 = $ 0 $\quad b_4 = $ 0

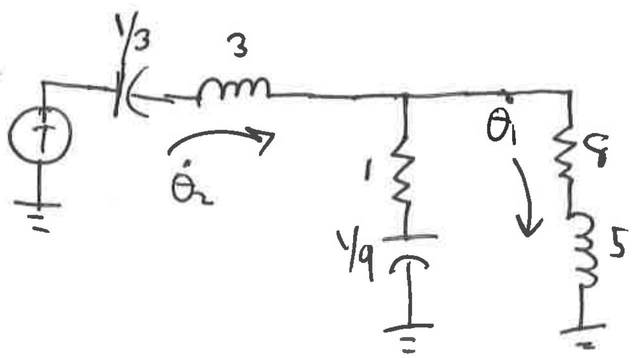


[8 pts] b) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to torque and current to rotational velocity. Input torque is $T(t)$.



redraw to make nodes clear





Key

c) Consider the system

$$G(s) = \frac{s+\beta}{s(s+1)(s+5)}\frac{5}{\beta}$$

[4 pts] i) Find $g(t)$ the inverse Laplace transform of $G(s)$.

$g(t) = ($ $\frac{5(1-\beta)}{4\beta}$ $e^{-t}+$ $\frac{\beta-5}{4\beta}$ $e^{-5t}+$ 1 $)u(t)$

$$\frac{5}{\beta}\frac{s+\beta}{s(s+1)(s+5)} = \frac{5}{\beta}\left[\frac{\beta/5}{s} + \frac{\frac{1-\beta}{4}}{s+1} + \frac{\frac{\beta-5}{20}}{s+5}\right]$$

$$\frac{5-5\beta}{4\beta} \qquad \frac{\beta-5}{4\beta}$$

[4 pts] ii) Find β such that at $t=0$ the contribution to $g(t)$ due to the pole at -5 is -5 times the contribution due the pole at -1.

$\beta =$ $5/6$

$$\frac{\frac{\beta-5}{20}}{\frac{1-\beta}{4}} = -5 = \frac{\frac{\beta-5}{5}}{1-\beta})$$

$$-5(1-\beta) = \frac{\beta-5}{5}$$

$$-25+25\beta = \beta-5$$

$$24\beta = 20$$

$$\beta = \frac{20}{24} = \frac{5}{6}$$

with this β,

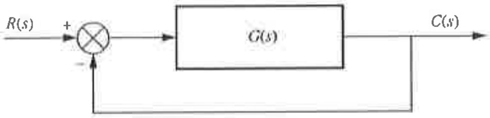
$g(t) = [1$

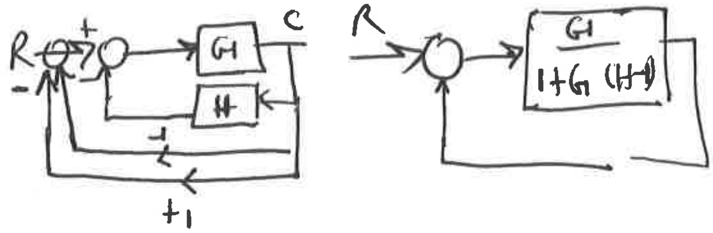
$$e^{-t}: \quad \frac{5(1-\frac{5}{6})}{4\cdot\frac{5}{6}} = \frac{1}{4}$$

$$e^{-5t}: \quad \frac{\frac{5}{6}-5}{4\cdot\frac{5}{6}} = \frac{1-6}{4} = \frac{-5}{4} \checkmark$$

Problem 2 Steady State Error (22 pts)

[6 pts] a) For the system at the bottom of the page, let $D(s) = 0$, $G_2(s) = 1$, $G_1(s) = \frac{k}{s+4}$, $H(s) = \frac{1}{s+1}$. That system can be redrawn in unity gain feedback form, as shown here. Determine the new open-loop transfer function $G(s)$ such that $\frac{C(s)}{R(s)}$ is the same in both cases.





$G(s) =$ ______

$$G = \frac{G_1}{1+G_1(H-1)} = \frac{k/s+4}{1+\frac{k}{s+4}\left(\frac{-s}{s+1}\right)} = \frac{k(s+1)}{(s+4)(s+1)-ks} \qquad H-1 = \frac{1-1-s}{s+1} = \frac{-s}{s+1}$$

[8 pts] b) For the system below, let $D(s) = 0$, $G_2(s) = 1$, $G_1(s) = \frac{k}{s+4}$, $H(s) = \frac{1}{s+1}$. Let $e(t) = r(t) - c(t)$. For $r(t) = u(t)$, a unit step, find the steady state expression for $e(t)$ for large t.

$e(t) =$ ______

$$C = \frac{G_1}{1+G_1H}R, \qquad R-C = \left(\frac{1+G_1H-G_1}{1+G_1H}\right)R$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,[R-C], \quad R = \frac{1}{s}$$

$$= \left(\frac{1+\frac{k}{(s+4)(s+1)}-\frac{k}{s+4}}{1+\frac{k}{s+4}\cdot\frac{1}{s+1}}\right)R$$

$$= \frac{4}{4+k}$$

$$= \left(\frac{(s+4)(s+1)+k-k(s+1)}{(s+4)(s+1)+k}\right)R$$

[8 pts] c) For the system below, let $H(s) = \frac{s+5}{s+10}$, $G_1(s) = \frac{k(s+1)}{s}$, and $G_2(s) = 1$. For $d(t) = tu(t)$, a unit ramp, and $r(t) = 0$, find the steady state expression for $c(t)$ for large t.

$c(t) = \underline{2/K.}$

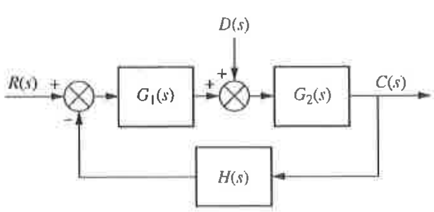
$$C = D - GHC, \quad C(1+G_1H) = D$$

$$\frac{C}{D} = \frac{1}{1+G_1H}, \quad C = \frac{1}{1+G_1H}\cdot D$$

$$\lim_{s\to 0} s\,C(s) = \frac{10}{5k}$$

$$= \frac{1}{1+\frac{(s+5)}{(s+10)}\cdot\frac{k(s+1)}{s}}\cdot\frac{1}{s^2}$$

$$= \frac{s(s+10)}{s(s+10)+(s+1)(s+5)k}\cdot\frac{1}{s^2}$$



Key.

Problem 3. Routh-Hurwitz (14 pts)

Given system with closed loop transfer function (assuming unity feedback)

$$T(s) = \frac{k}{s^4 + 4s^3 + 6s^2 + 4s + 1 + k}$$

[10 pts] a. Using the Routh-Hurwitz table, find the range of k for which the closed loop system is stable.

$\underline{-1} < k < \underline{4}$

$s^4 \quad 1 \quad 6 \quad 1+k$

$s^3 \quad 4 \quad 4 \quad 0$

$s^2 \quad \dfrac{-\begin{vmatrix} 1 & 6 \\ 4 & 4 \end{vmatrix}}{4} = 5 \quad \dfrac{-\begin{vmatrix} 1 & 1+k \\ 4 & 0 \end{vmatrix}}{4} = 1+k \quad 0$

$s^1 \quad \dfrac{-\begin{vmatrix} 4 & 4 \\ 5 & 1+k \end{vmatrix}}{5} = \dfrac{20-4-4k}{5} = \dfrac{16-4k}{5} \quad -\begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} = 0 \quad 0$

$s^0 = \dfrac{-\begin{vmatrix} 5 & 1+k \\ 16-4k & 0 \end{vmatrix}}{16-4k} = 1+k$

$\Rightarrow$ 1) $1+k > 0$, $k > -1$

2) $16-4k > 0$, $16 > 4k$, $k < 4$

[4 pts] b. For the positive value of k found above, find the pair of closed loop poles on the imaginary axis. (Show work).

$s = \pm j\omega_o = \pm \underline{j}$

$k=4$ s^2 row is 5 5

thus $5s^2+5=0$

$s^2+1=0$

or $s=\pm j$

Key.

Problem 4. Root Locus Plotting (27 pts)
Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s+4)}{(s^2+2s+2)(s^2+4s+8)}$$

$s = \frac{-1 \pm \sqrt{4-8}}{2} = -1 \pm j$

$s = \frac{-2 \pm \sqrt{16-32}}{2} = -2 \pm 2j$

For the root locus $(1 + kG(s) = 0)$ with $k > 0$:

[1 pts] a) Determine the number of branches of the root locus = 4

[2 pts] b) Determine the locus of poles on the real axis $-\infty < \sigma < -4$

[3 pts] c) Determine the angles for each asymptote: $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ $\qquad \theta = \frac{(2l+1)\pi}{4-1}$

[4 pts] d) determine the real axis intercept for the asymptotes $s =$____

$$\sigma_{asymptote} = \frac{\sum poles - \sum zeros}{n-m} = \frac{-1-1-2-2-(-4)}{3} = \frac{-2}{3}$$

[6 pts] e) Determine the angle of departure for the root locus for the pole at $s = -2+2j =$ $+252^\circ$

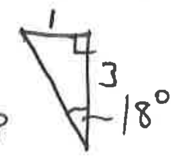
angles to $-2+2j$:

$\measuredangle s+4 = +45^\circ$ $\qquad -\measuredangle(s+1+j) = -135^\circ$ $\qquad$ total $= -135^\circ$

$-\measuredangle s+2-2j = -90^\circ$ $\qquad -\measuredangle(s+1-j) = -108^\circ$ $\qquad -153^\circ$

-288°

$\boxed{-252} = -548$



[6 pts] f) Estimate the value of k for which the closed loop system has a pole at $s \approx +2.5j$.

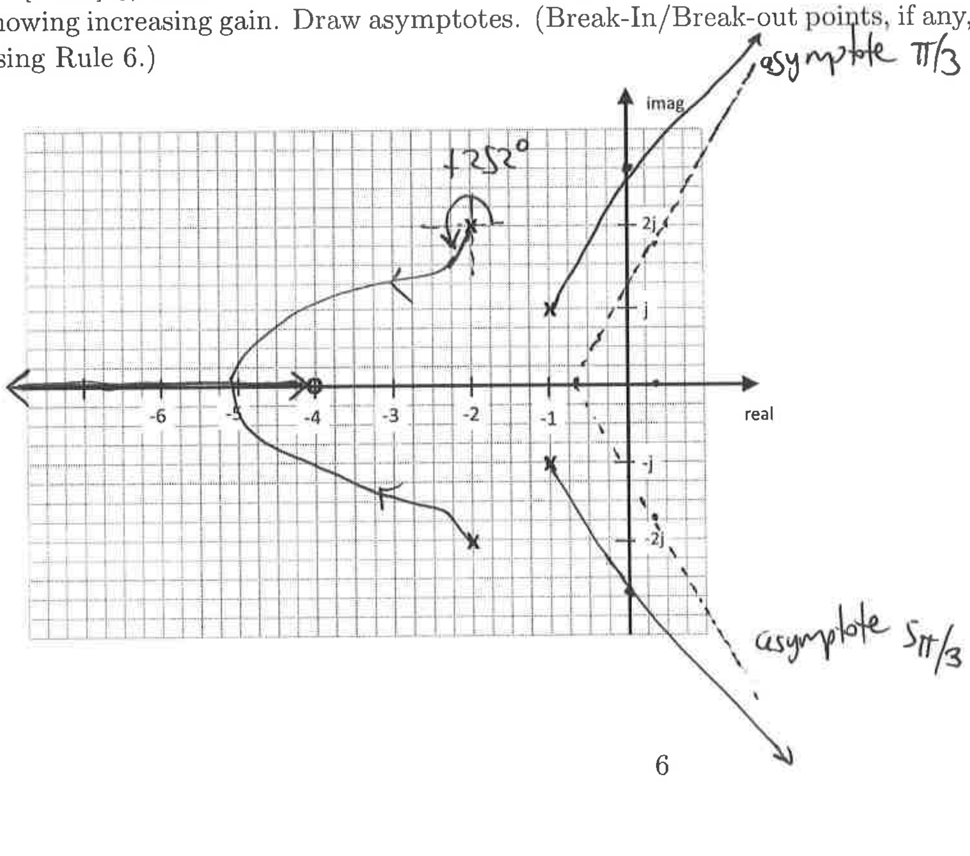
$k =$ 15

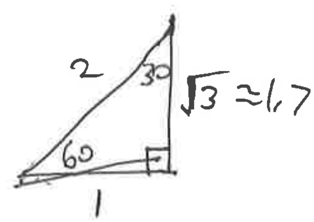
$$|G|_{s=2.5j} \approx \frac{\sqrt{16+6.3}}{\sqrt{1+2.3}\sqrt{1+10}\sqrt{4+1/4}\sqrt{4+20}} \approx \frac{\sqrt{20}}{6 \cdot 10} \approx \frac{4}{60} \approx 1/15$$

$\sqrt{36} \quad \sqrt{100}$

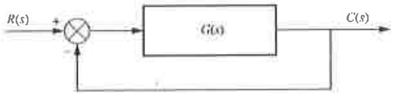
$|kG| = 1 \qquad k \approx \frac{1}{|G|} =$

[5 pts] g) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. Draw asymptotes. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)





Problem 5. Root Locus Compensation (15 pts) Key.



Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{s+4}{(s+1)^2(s+5)^2}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a PD compensation of the form $G_1(s) = k\frac{s+z_c}{1}$. The closed loop system, using unity gain feedback and the PD controller, should have a pair of poles at $p = -2 \pm j$.

[4 pts] a. Show that for closed loop poles p, that the angle contribution contribution from $G_3(p)$ is $\approx -280°$.

angles to $-2+j$:

$\angle s+4 = +27°$

$2\{-\angle s+1\} = -2(135) = -270°$

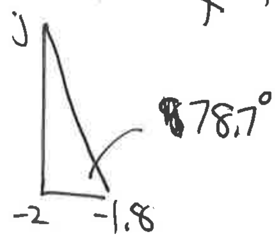
$-2\angle s+5 = -2(18.4) = -37°$

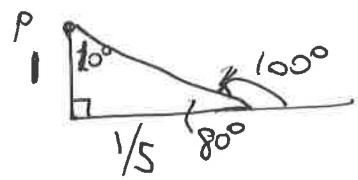
$\Sigma \text{angles} = -270 - 37 + 27 \simeq \boxed{-280°}$

[9 pts] b. Find z_c to within ± 0.1 such that p is approximately on the root locus, within $\pm$ 5 degrees. (Show work.)

	value	angle
z_c	1.8	+101 °

$\angle s+z_c$ must contribute $+100°$

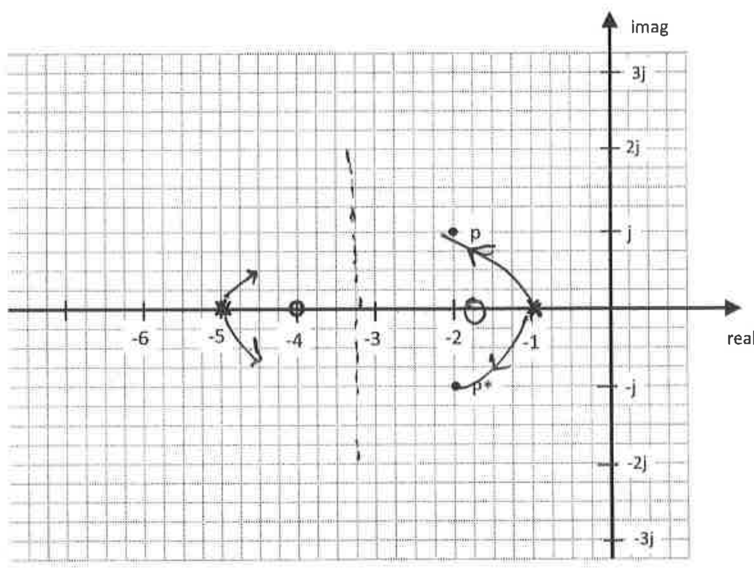




$\tan^{-1}\frac{1}{5} = 11.3°$

[2 pts] c. Briefly explain why or why not the closed loop system step response can be characterized by a dominant pole pair assumption, (with poles at $p = -2 \pm j$). Depends on partial fraction expansion, but zero at -1.8 is not cancelled. Also C.L.P. from -5 are not 5x farther, so violates basic assumptions.

(Pole-Zero plot below for scratch work. It will not be graded).



asymptotes $\pm\frac{\pi}{2}$

asymptote intercept

$$\frac{\Sigma p - \Sigma z}{4-2} = \frac{-1-1-5-5-(-4)-(-1.8)}{2}$$

$$= \frac{-6.2}{2} = -3.1$$

page for scratch work