Physics 137B, Spring 2016 Midterm 1
Good luck!

1 Little calculation needed

a. [**5 points**] Two neutrons (spin 1/2) are in the ground state of a harmonic oscillator potential. Neglect any interactions between them. What can you say about the orientation of their spin?

b. [**5 points**] What happens to the expectation value $\langle x^2 \rangle$ averaged over all particles when a third neutron is added, assuming it takes the lowest available energy level?

c. [**5 points**] A particle is in the ground state of a harmonic oscillator when a potential $H' = \alpha x$ (a constant weak force) is applied. Does the energy of the system go up or down?

d. [**5 points**] Calculate the correction $\psi_0^{(1)}$ to the ground state in first-order perturbation theory for the system in part c.

2 Yukawa potential

Consider a hydrogen atom in a state $|n, l, m\rangle$, neglecting fine structure or hyperfine structure. It has been suggested[1] that the Higgs field leads to an additional potential between the proton and the electron of the form

$$V(r) = A\frac{e^{-r/\lambda}}{r},$$

where A and λ are constants. This Yukawa potential is typical for interactions mediated by massive particles.

a. [**10 points**] Use first-order, nondegenerate perturbation theory to calculate the energy change for the ground state $|1, 0, 0\rangle$.

b. [**5 points**] Justify the use of nondegenerate perturbation theory in this problem even for excited states of hydrogen, despite their degeneracy.

[1] C. Delaunay, R. Ozeri, G. Perez, and Y. Soreq, Probing The Atomic Higgs Force. e-print: arXiv:1601.05087

3 Degenerate perturbation theory

Consider a system with three eigenstates $|\psi_j\rangle$ ($j = 1, 2, 3$). The Hamiltonian of this system can be written as

$$\langle\psi_i|\, H_0\, |\psi_j\rangle = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_{23} & 0 \\ 0 & 0 & \epsilon_{23} \end{bmatrix}$$

where $\epsilon_1 < \epsilon_{23}$ and $|\psi_2\rangle$ and $|\psi_3\rangle$ are degenerate in energy. The system is now perturbed such that the new Hamiltonian can be written as $H = H_0 + H_1$ where

$$\langle\psi_i|\, H_1\, |\psi_j\rangle = \begin{bmatrix} \tau & \Delta & -i\sigma \\ \Delta & \delta & i\gamma \\ i\sigma & -i\gamma & \delta \end{bmatrix}$$

and Δ, δ, σ, γ, and τ are all real.

(a) [**5 points**] Determine the correct 0th order wave functions for the two degenerate states ($j = 2, 3$) that you would use for degenerate perturbation theory. Denote them $|\psi_{23a}\rangle$ and $|\psi_{23b}\rangle$.

(b) [**5 points**] Determine the 1st correction to the energy of $|\psi_{23a}\rangle$.

(c) [**5 points**] Determine the 2nd correction to the energy of $|\psi_{23a}\rangle$.

Useful equations

Hydrogen ground state wave function

$$\psi_{100}(r, \theta\phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}.$$

An integral

$$\int r e^{-\alpha r} dr = -\frac{1 + \alpha r}{\alpha^2} e^{-\alpha r}$$

Harmonic oscillator:

$$a = \frac{\omega m x + ip}{\sqrt{2\omega m \hbar}}, \quad a^\dagger = \frac{\omega m x - ip}{\sqrt{2\omega m \hbar}}, \quad [a, a^\dagger] = 1,$$

$$H = \hbar\omega(\hat{n} + 1/2), \quad \hat{n} = a^\dagger a, \quad \hat{n}|n\rangle = n|n\rangle,$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad a|n\rangle = \sqrt{n}|n-1\rangle.$$