MT 2

Phys 137B, Spring 2013
(Dated: April 4, 2013)

v2: Second line of 5a: corrected $2/3 \to 2/9$

1) **20 points** Using the WKB method, find the bound state energies E_n of a particle of mass m in a V-shaped potential well

$$V(x) = \begin{cases} -V_0(1 - |x/a|) & -a < x < a \\ V(x) = 0 & \text{otherwise.} \end{cases} \tag{1}$$

$V_0 > 0$ and $a > 0$ are constants (Fig. 1). What's the energy of the highest bound state?

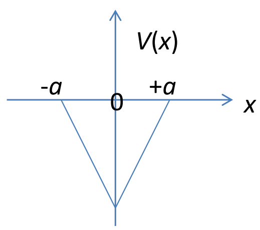

FIG. 1. V-shaped potential.

2) **30 points** Using the WKB method, find the bound state energies for a particle of mass m bouncing up and down atop a surface in the gravitational field. For a vertical coordinate $z > 0$, let the potential be purely gravitational: $V = mgz$, where g is the acceleration of free fall. For $z < 0$, assume a repulsive quadratic potential $V = \frac{1}{2}kz^2$, where $k > 0$ is a constant. All in all,

$$V = \begin{cases} mgz & z > 0 \\ +\frac{1}{2}kz^2 & z \leqslant 0 \end{cases} \tag{2}$$

(See Fig. 2). Calculate the bound energy levels E_n and the corresponding WKB wave functions. Give an explicit result for the first two E_n as frequencies (in Hz) $\nu_n = E_n/h$ for $g = 9.8\text{m/s}^2$, $m = 1.66 \times 10^{-27}$ kg (approximately one atomic mass unit), and $k = 1 \times 10^{-19}$ N/m.

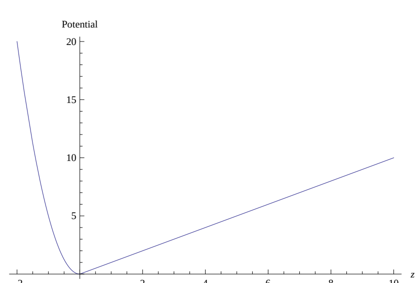

FIG. 2. Potential of the quantum bouncer.

3) A mass m on a spring of spring constant k forms a one-dimensional harmonic oscillator with resonance frequency $\omega_0 = \sqrt{k/m}$ (Fig. 3). The system is initially in the ground state $|0\rangle$. Which of the following strategies will allow bringing the oscillator into the state $|1\rangle$? In each case, calculate the probability of finding the system in the state $|1\rangle$ at a time T after the perturbation has initially been switched on. Use time-dependent perturbation theory. It's probably a good idea to use ladder operators.

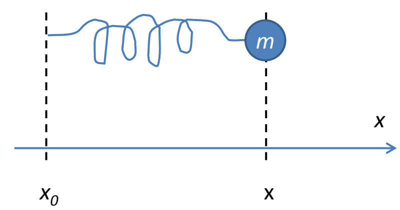

FIG. 3. Mass on a spring. Except for part (b), assume that the spring is attached to a rigid point on the left hand side, so that $x_0 = 0$.

(a) **10 points** Application of a small force $F(t) = F_0 \cos \omega t$ to the particle

(b) **10 points** Modulating the point x_0 (see Fig. 3) at which the spring is attached according to $x_0 = A \cos \omega t$ with a small amplitude A.

(c) **10 points** Modulating the spring constant $k \to k_0 + k' \cos \omega t$, where $k' > 0$ is a constant. You may assume that $k' \ll k$ if that helps.

(d) **10 points** Modulating the mass according to $m = m_0 + m' \cos \omega t$. You may assume that $m' \ll m$.

(e) **10 points** Application of a sudden force $F(t) = F_0 \delta(t)$.

4) Consider a two-level system with an energy difference $\hbar\omega_0$ driven sinusoidally by a perturbation $H' = V_{ab} \cos \omega t$, where we denote $V_{ab} = \hbar\Omega_R$ and $\delta = \omega_0 - \omega$. Assume $|\delta| \ll \omega_0$. The system is in the ground state before a perturbation is applied for a time T. You're trying to bring it to the excited state with at least 2/9 probability, i.e., $|c_b|^2 \geqslant 2/9$

(a) **5 points** What is the maximum value of δ that allows reaching $|c_b|^2 \geqslant 2/9$, provided that for each δ, the optimum T can be chosen? **5 bonus points**: Illustrate your answer using the Bloch sphere. (Beware that the definition of Ω_R here and in my Bloch sphere notes - used there for convenience in the derivation - disagree by a factor of 2)

(b) **5 points** What is the maximum value of δ, provided that T is fixed at the value that leads to $|c_b|^2 = 1$ for $\delta = 0$?