Midterm 2

Phys 137B
(Dated: April 14, 2016)

Hints: several problems may be solved without calculation, or very little calculation. If you choose to do so, justify your answer specifically by the principles/theorems that you are following.

1. **Rabi problem** Here is the (exact) Rabi solution for a two-level system with a level splitting of $\hbar\omega_0$ driven by perturbation of the form $H_{ab} = (V_{ab}/2)e^{i\omega t}$, $H_{ba} = (H_{ab})^*$. The amplitudes for the two states are $c_a(t), c_b(t)$.

$$c_a(t) = \left[\cos\left(\frac{\Omega t}{2}\right) + i\frac{\delta}{\Omega}\sin\left(\frac{\Omega t}{2}\right)\right]e^{-i\delta t/2}, \qquad (1)$$

$$c_b(t) = -i\frac{\Omega_R}{\Omega}\sin\left(\frac{\Omega t}{2}\right)e^{i\delta t/2}, \qquad (2)$$

where $\Omega = \sqrt{\Omega_R^2 + \delta^2}$, $\Omega_R = V_{ab}/\hbar$ is the *Rabi frequency* and $\delta = \omega_0 - \omega$ is the detuning.

a) **10 points** What are the initial conditions $c_a(0), c_b(0)$ of this solution?

b) **10 points** Find the solution for initial conditions $c_a(0) = 0, c_b(0) = 1$. Hint: you don't need to solve the differential equations for the two-level system from scratch. Instead, modify the above solution.

c) **10 points** Assume $\delta = 0$. Starting from the differential equations

$$\dot{c}_a = -\frac{i}{\hbar}H_{ab}e^{-i\omega_0 t}c_b, \quad \dot{c}_b = -\frac{i}{\hbar}H_{ba}e^{i\omega_0 t}c_a \qquad (3)$$

and show that Eqns. (1, 2) are indeed an exact solution to the Schrödinger equation.

2) **10 points** Use the variational principle to show that first-order non-degenerate perturbation theory overestimates (and never underestimates) the ground state energy.

3) A mass m on a spring of spring constant k forms a one-dimensional harmonic oscillator with resonance frequency $\omega_0 = \sqrt{k/m}$ (Fig. 3). The system is initially in the ground state $|0\rangle$. Which of the following strategies will allow bringing the oscillator into the state $|1\rangle$? In each case, calculate the probability of finding the system in the state $|1\rangle$ at a time T after the perturbation has initially been switched on.

Hints. Use time-dependent perturbation theory and ignore the higher states $|2\rangle, |3\rangle, \ldots$. It's probably a good idea to use ladder operators. In all of the following, use the notation $\delta = \omega_0 - \omega$. You may assume $|\delta| \ll |\omega_0|$.

(a) **10 points** Modulating the point x_0 (see Fig. 1) at which the spring is attached according to $x_0 = A\cos\omega t$ with a small amplitude A.

(b) **10 points** Modulating the mass according to $m = m_0 + m'\cos\omega t$. You may assume that $m' \ll m_0$.

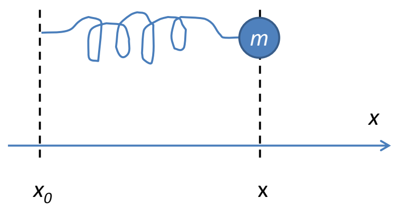

FIG. 1. Mass on a spring. Except for part (b), assume that the spring is attached to a rigid point on the left hand side, so that $x_0 = 0$.

(c) **10 points** Application of an impulsive force $F(t) = p_0\delta(t)$.

4. **Geometric phase (10 points)** A particle of mass m is in the second excited state $n = 2$ of a harmonic oscillator potential $V = \frac{1}{2}m\omega^2x^2$. What is the geometric phase γ_2 that the state accumulates when ω is adiabatically ramped down to half its initial value?

End of the exam. There are 80 points total.