UNIVERSITY OF CALIFORNIA, BERKELEY	**Last name:**	________________
MECHANICAL ENGINEERING	**First name:**	________________
ME 106, FLUID MECHANICS	**Student ID:**	________________
ODK/MIDTERM 3, FALL 2015	**Discussion:**	________________

Notes:

- You solution procedure should be legible and complete for full credit (use scratch paper as needed).

Question	Grade
1	
2	
3	
Total:	

1. Consider a drop of liquid falling through air. The drag force can cause the drop to break up. Suppose the force (F_d) to break up the drop depends on the drop's density (ρ), speed (V), diameter (D) and surface tension (σ). (Recall σ has dimensions force per distance.)

   (a) Determine the dimensionless parameters characterizing this problem. Choose V, D, ρ as the repeating variables.

   (b) Invoke the Buckingham Pi theorem to write a general functional relationship between the obtained parameters.

   (c) Suppose $F_d = 0.01\,N$ for a water drop with diameter D translating with speed $V_1 = 3m/s$. Determine F_d for a drop of the same size translating at twice the speed $V_2 = 6\,m/s$. (ρ is constant)

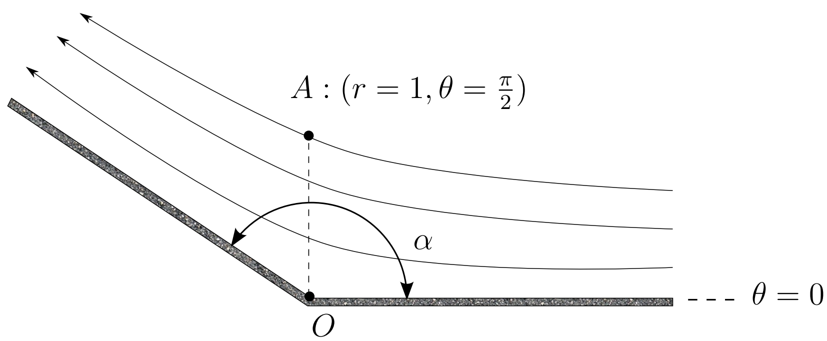
2. 2D steady inviscid flow near a corner can be modeled with the potential function $\phi(r,\theta) = r^n \cos(n\theta)$ in polar coordinates, where $n = \frac{\pi}{\alpha} > 1$, $r \geq 0$ and $0 \leq \theta \leq \alpha$. (Note: Relevant relations in cylindrical coordinates are given in the equation sheet).

   (a) Find the velocity vector field $\vec{V} = u_r\vec{e}_r + u_\theta\vec{e}_\theta$. Locate the stagnation point.

   (b) Is this flow irrotational? (Justify)

   (c) Is this flow incompressible? (Justify)

   (d) If the stagnation pressure P_O is given, find the pressure P_A at point A. Assume density ρ is given and gravity is negligible.



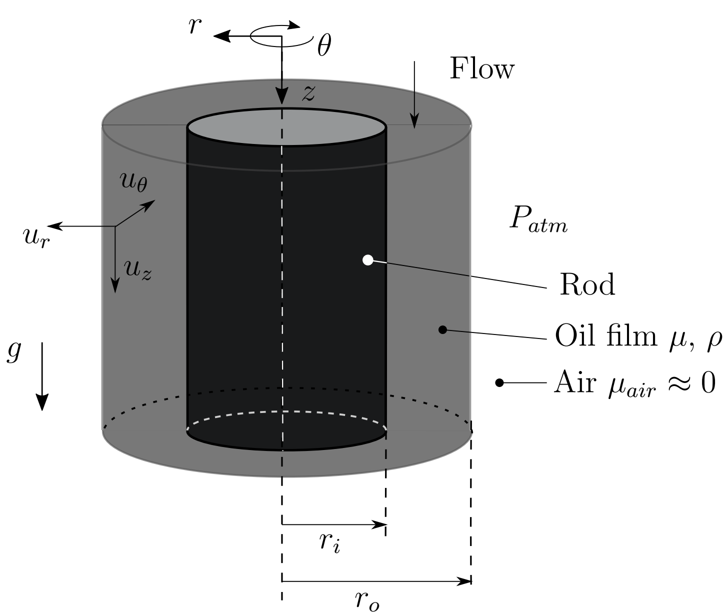
3. A viscous oil film of outer radius r_o uniformly drains down the side of a stationary vertical rod of radius r_i due to gravity. Assume the flow is steady and incompressible. The air surrounding the oil imparts no shear stress. The rod can be considered infinitely long.

   (a) Write down (justified) assumptions that enable you to reduce the continuity equation and Navier-Stokes equation. (Or write down "as needed" when completing parts 3b and 3c below).

   (b) Using the continuity equation in cylindrical coordinates (see equation sheet), show that the flow is fully developed (i.e., $\frac{\partial u_z}{\partial z} = 0$). Density is constant.

   (c) Using Navier-Stokes equation in cylindrical coordinates (see equation sheet), simplify each component equation. Determine what r and θ equations imply regarding pressure, and show the z equation reduces to an ODE. You do not need to copy the full equations; rather you can simplify the ones on the equation sheet and write the result here (terms set to zero must match your assumptions above).

   (d) Integrate the ODE derived above to obtain an expression for $u_z(r)$. State appropriate boundary condition(s) to solve for integration constants. (You do not need to solve for them but can if you have time.)



Summary of Equations:

Stream function $$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \textbf{(6.37)}$$

Euler's equations of motion
$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \qquad \textbf{(6.51a)}$$
$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \qquad \textbf{(6.51b)}$$
$$\rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) \qquad \textbf{(6.51c)}$$

Velocity potential $$\mathbf{V} = \nabla\phi \qquad \textbf{(6.65)}$$

Laplace's equation $$\nabla^2\phi = 0 \qquad \textbf{(6.66)}$$

Uniform potential flow $$\phi = U(x\cos\alpha + y\sin\alpha) \qquad \psi = U(y\cos\alpha - x\sin\alpha) \qquad u = U\cos\alpha,\; v = U\sin\alpha$$

Source and sink $$\phi = \frac{m}{2\pi}\ln r \qquad \psi = \frac{m}{2\pi}\theta \qquad v_r = \frac{m}{2\pi r},\; v_\theta = 0$$

Vortex $$\phi = \frac{\Gamma}{2\pi}\theta \qquad \psi = -\frac{\Gamma}{2\pi}\ln r \qquad v_r = 0,\; v_\theta = \frac{\Gamma}{2\pi r}$$

Doublet $$\phi = \frac{K\cos\theta}{r} \qquad \psi = -\frac{K\sin\theta}{r} \qquad v_r = -\frac{K\cos\theta}{r^2},\; v_\theta = -\frac{K\cos\theta}{r^2}$$

The Navier–Stokes equations

(x direction)
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad \textbf{(6.127a)}$$

(y direction)
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \qquad \textbf{(6.127b)}$$

(z direction)
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \qquad \textbf{(6.127c)}$$

Cylindrical Coordinate relations:

Potential function:
$$u_r = \frac{\partial \phi}{\partial r}, \qquad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

Stream function:
$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = -\frac{\partial \psi}{\partial r}.$$

Gradient:
$$\nabla\phi = \frac{\partial \phi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\vec{e}_\theta + \frac{\partial \phi}{\partial z}\vec{e}_z.$$

Divergence:
$$\nabla\cdot\vec{V} = \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}.$$

Curl:
$$\nabla\times\vec{V} = \frac{1}{r}\begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ u_r & r u_\theta & u_z \end{vmatrix}.$$

Continuity equation in cylindrical coordinates:
$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0.$$

Navier-Stokes equation in cylindrical coordinates:
$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + g_r + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right],$$
$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + g_\theta + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right],$$
$$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right].$$