Solutions to Midterm 2

Problem 1. Set up an absolute (x, y) frame with the x-axis directed horizontally to the left. For the system of rod and two balls,

$$\sum F_x = m\bar{a}_x$$

$$\Rightarrow \quad 12 = \left(\frac{4+2}{32.2}\right)\bar{a}$$

$$\Rightarrow \quad \bar{a} = 64.4 \text{ ft/sec}^2$$

The distance between mass center G and the 2-lb ball is given by

$$4(10 - b) = 2b$$

$$\Rightarrow \quad b = \frac{20}{3} \text{ in} = \frac{5}{9} \text{ ft}$$

The angular position may be measured by the angle θ between the rod and the vertical.

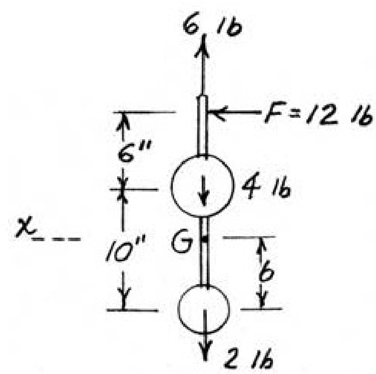
$$H_G = (H_G)_{\text{rel}} = \sum m_i r_i^2 \dot{\theta} = \frac{4}{32.2}\left(\frac{10-b}{12}\right)^2 \dot{\theta} + \frac{2}{32.2}\left(\frac{b}{12}\right)^2 \dot{\theta} = 0.0288\dot{\theta}$$

For the system,

$$\sum M_G = \dot{H}_G$$

$$\Rightarrow \quad F\left[\frac{6+(10-b)}{12}\right] = 0.0288\ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} = 325 \text{ rad/sec}^2 \qquad \text{CCW}$$



Problem 2. Attach an absolute (x, y) frame to O with the x-axis directed to the right. Point A moves in a circle about O and point B in a circle about C. For A, B on plate ABD,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$\Rightarrow \quad v_A\mathbf{j} = v_B\mathbf{i} + \omega_{AB}\mathbf{k} \times (\mathbf{r}_A - \mathbf{r}_B) = v_B\mathbf{i} + (-3\mathbf{k}) \times [3\mathbf{i} - (7\mathbf{i} + 3\mathbf{j})]$$

$$\Rightarrow \quad v_A\mathbf{j} = v_B\mathbf{i} + (-3\mathbf{k}) \times (-4\mathbf{i} - 3\mathbf{j})$$

Equate **i** coefficients,

$$v_B = 9 \text{ in/sec}$$

The velocity v_B is directed to the right. Since B moves in a circle about fixed point C,

$$\omega_{BC} = \frac{v_B}{BC} = \frac{9}{3} = 3 \text{ rad/sec} \qquad \text{CW}$$

3. $\underline{v}_A = -10\underline{i} + 12\underline{j}$, $\underline{e}_1 = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$

$\underline{e}_2 = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j}$

a)

$\{\underline{e}_1, \underline{e}_2, k\} \rightarrow$ rotating axes

$$\underline{v}_A = \underline{v}_{rel} + \underline{\omega} \times \underline{r}$$

$$-10\underline{i} + 12\underline{j} = u\,\underline{e}_2 + \omega\,\underline{k} \times (L\,\underline{e}_1 + d\,\underline{e}_2)$$

$$-10\underline{i} + 12\underline{j} = u\,\underline{e}_2 + 2\omega\,\underline{e}_2 - \omega\,\underline{e}_1$$

$$-10\,\underline{i}\cdot\underline{e}_1 + 12\,\underline{j}\cdot\underline{e}_1 = -\omega \qquad (1)$$

$$-10\,\underline{i}\cdot\underline{e}_2 + 12\,\underline{j}\cdot\underline{e}_2 = u + 2\omega \qquad (2)$$

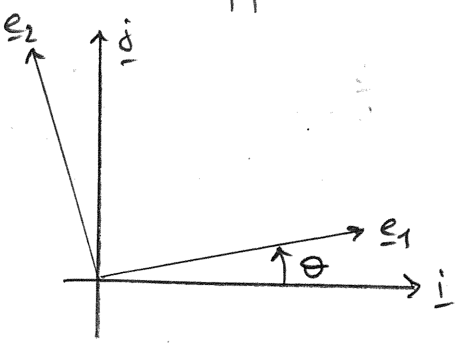
$(1) \rightarrow \quad -10\cos 30^\circ + 12\sin 30^\circ = -\omega \Rightarrow \omega = 2.6603$ rad/s

$(2) \rightarrow \quad +10\sin 30^\circ + 12\cos 30^\circ = u + 2(2.6603) \Rightarrow u = 10.0717$ m/s

b)

$$\underline{a}_{cor.} = 2\,\underline{\omega} \times \underline{v}_{rel}$$
$$= (2)\; 2.6603\,\underline{k} \times 10.0717\,\underline{e}_2$$
$$= -53.5875\,\underline{e}_1 \quad m/s^2$$

It appears due to angular velocity of basis.



$\dot{\underline{e}}_1 \neq 0 \quad \dot{\underline{e}}_2 \neq 0$

$\dot{\underline{e}}_1 = \underline{\omega} \times \underline{e}_1$

$\dot{\underline{e}}_2 = \underline{\omega} \times \underline{e}_2$

$$\underline{r} = r_1\,\underline{e}_1 + r_2\,\underline{e}_2$$

$$\underline{v} = \dot{r}_1\,\underline{e}_1 + \dot{r}_2\,\underline{e}_2 + \dot{\theta}\,\underline{k} \times (r_1\,\underline{e}_1 + r_2\,\underline{e}_2)$$
$$= \underline{v}_{rel} + \underline{\omega} \times \underline{r}$$

$$\underline{a} = \dot{\underline{v}}_{rel} + \underline{\alpha} \times \underline{r} + \underline{\omega} \times \dot{\underline{v}}$$
$$= \underline{a}_{rel} + \underline{\omega} \times \underline{v}_{rel} + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{v}_{rel} + \underline{\omega} \times \underline{r})$$
$$= \underline{a}_{rel} + \underline{\underline{2\,\underline{\omega} \times \underline{v}_{rel}}} + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$