MIDTERM II: MATH H53

2017. 3. 23.
8:10 AM-9:20 AM

Student Name: (First) ____________________ (Last) ____________________

Signature: ____________________

Berkeley e-mail address : ____________________ @ berkeley.edu

1. Determine whether the following function is continuous at $(0,0)$ or not.

(1) (10 points)

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+2y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Answer:

(2) (10 points)

$$f(x,y) = \begin{cases} xy \sin \frac{1}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Answer:

2. Consider the curve $\mathbf{r}(t) = \left\langle t+1,\ \frac{1}{2}t^2,\ 3 \right\rangle$, $t \in \mathbb{R}$.

(1) (6 points) Find the curvature of the curve at $\mathbf{r}(0)$.

Answer:

(2) (14 points) Find the principal unit normal vector and unit binormal vector to the curve at $\mathbf{r}(0)$.

Answer:

(3) (8 points) Find the osculating plane at $\mathbf{r}(0)$.

Answer:

(4) (12 points) Find the osculating circle at $\mathbf{r}(0)$.

Answer:

3. (15 points) Let $g(x, y)$ and $h(x, y)$ are smooth functions and satisfy

$$g(0,0) = h(0,0) = 1 \ ,$$
$$\nabla g(0,0) = \langle 1, 2 \rangle \ , \ \nabla h(0,0) = \langle -1, 4 \rangle \ .$$

Let $f(x, y)$ be a smooth function satisfying $\nabla f(1, 1) = \langle 1, 2 \rangle$. Find gradient of the function $p(x, y) = f(g(x, y), h(x, y))$ at $(x, y) = (0, 0)$.

Answer:

4. (20 points) Let S be a surface described by the equation $x^2 + y^2 + z^2 + kxyz = 2$ for some constant k. Furthermore, assume that the tangent plane to the surface S at $(1, 1, 0)$ is given by $x + y + 2z = 2$. Find the value of k and find the tangent line to the curve $2x^2 + k \sin(xy) = 2$ at $(1, 0)$.

Answer:

5. (20 points) Find the maximum and minimum of the function $f(x,y) = x^2 + 2y^2 - 2xy - 4y$ on

$$\{(x,y) : -2 \leq x \leq 2\ , \quad -2 \leq y \leq 2\}\ .$$

Answer:

6. (20 points) Consider a function $f(x, y, z) = x^3 + y^3 + z^3$. Find the maximum and minimum of $f(x, y, z)$ subject to the condition $x^2 + y^2 + z^2 = 3$.

Answer:

7. (15 points) Let D be a region in $\mathbb{R}^2$ defined by

$$D = \{(x, y) : x^2 + 2y^2 \leq 4 \ x \geq 0\} \ ,$$

and let f be a function on D defined by $f(x, y) = xy^2$. Evaluate $\iint_D f(x, y) dA$.

Answer:

8. (Bonus Problem, +20 points) Find the maximum and minimum of $f(x, y, z) = x^3 + y^3 + z^3$ subject to two conditions $x^2 + y^2 + z^2 = 3$ and $x + y + z = 0$.

Answer: