MIDTERM I: MATH H53

2017. 2. 14.
8:10 AM-9:10 AM

Student Name: (First) ____________________ (Last) ____________________

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1. Consider the curve C corresponding to the following parametric equation:

$$x = \cos t + \sin t + 1\ ,\quad y = \cos t - \sin t + 2\ \ ;\ 0 \le t \le 2\pi\ .$$

(1) [10 points] Find the equation of the tangent line to the curve C at (2, 1).

Answer:

(2) [10 points] Find the length of the curve C.

Answer:

2. [10 points] Express the following curve by the polar equation $r = f(\theta)$:

$$x^4 + y^4 = x^3 + y^3 \ .$$

Answer:

3. [15 points] Find two foci of the following conic curve:

$$x^2 - y^2 + 2x + 2y - 1 = 0 \ .$$

Answer:

4. Consider the curve C described by the following polar equation:

$$r = e^{2\theta} \ ; \ \ 0 \leq \theta \leq 2\pi \ .$$

(1) [10 points] Find the equation of the tangent line to the curve C at $(r, \theta) = (e^{2\pi}, \pi)$.

Answer:

(2) [10 points] Find the length of the curve C.

Answer:

(3) [10 points] Find the area of the region enclosed by the curve C and the line segment connecting $(r, \theta) = (1, 0)$ and $(r, \theta) = (e^{4\pi}, 2\pi)$.

Answer:

5. Consider two curves C_1 and C_2 determined by the following parametric equations:

$$C_1 : \quad x = t^3 \ ; \ y = t + 1 \ ; \ t \in \mathbb{R} \, ,$$
$$C_2 : \quad x = t^3 \ ; \ y = t^2 + t \ ; \ t \in \mathbb{R} \, .$$

(1) [7 points] Find two intersection points between C_1 and C_2.

Answer:

(2) [18 points] Find the area of the region enclosed by both C_1 and C_2.

Answer:

6. Consider two lines ℓ_1 and ℓ_2 given by the following vector equations:

$$\ell_1 : \qquad \langle -1,\, 2,\, 1\rangle + t\,\langle 2,\, -1,\, 0\rangle \quad ;\ t \in \mathbb{R}$$
$$\ell_2 : \qquad \langle 3,\, 3,\, 2\rangle + t\,\langle 2,\, 2,\, 1\rangle \quad ;\ t \in \mathbb{R}\ .$$

(1) [6 points] Find the intersection point of two lines ℓ_1 and ℓ_2.

Answer:

(2) [14 points] Let $0 \le \theta < \pi$ be the angle between two lines ℓ_1 and ℓ_2. Find $\cos\theta$.

Answer:

(3) [20 points] Find the equation of the plane P which contains both ℓ_1 and ℓ_2.

Answer:

(4) [10 points] Find the distance between the point (3, 3, 4) and the plane P.

Answer:

7. [Bonus problem, +20 points] Suppose that two vectors $\vec{a}$ and $\vec{b}$ satisfy

$$|\vec{a}| = 1\ ,\quad |\vec{a} + \vec{b}| = 2 \ \text{ and } \ |\vec{a} + 2\vec{b}| = 4\ .$$

Find $|\vec{b}|$.

Answer: