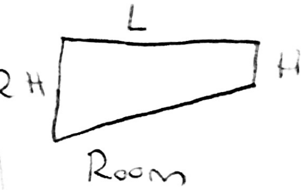
PHYS 7B Spring 2017, Bale Solution Midterm 1

1)

a) $\boxed{M_{air} = \rho_{Air} V_{room}}$ $\boxed{V_{room} = \frac{3}{2} L^2 H}$, $L = 15\,m$, $H = 3\,m$



$$V_{room} = \frac{3}{2}(15^2 \cdot 3)$$

$$\boxed{V_{room} = \frac{45^2}{2} \simeq 1000\,m^3}$$

$$\Rightarrow \boxed{M_{air} \simeq \frac{1\,kg}{m^3} \cdot 1000\,m^3 = 1000\,kg}$$

b) change in # moles for room w/ constant P, V, when T is changed:

$$\Delta n = n_f - n_i = \frac{PV_{room}}{RT_f} - \frac{PV_{room}}{RT_i} = \boxed{\frac{PV_{room}}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right) = \Delta n}$$

$$\boxed{M_{leaves} = \Delta n \times m_{air}}, \quad m_{air} \simeq m_{N_2} = \frac{0.03\,kg}{1\,mol}$$

molar mass of N_2

molar mass of air

$$\Delta n \simeq \frac{10^5 \frac{N}{m^2} \cdot 1000\,m^3}{8 \frac{J}{mol \cdot K}} \underbrace{\left(\frac{1}{288} - \frac{1}{293}\right)}_{} \simeq \frac{10^8}{8}(mol \cdot K)\left(\frac{1}{2} \times 10^{-4}\right) \approx \frac{10^4}{16}$$

$$\boxed{\Delta n \approx 5 \times 10^2\,mol}$$

$$\frac{1}{288}\left(1 - \frac{288}{293}\right)$$

$$\frac{1}{288}\left(\frac{5}{293}\right) \approx \frac{5}{300^2} \approx \frac{5}{9 \times 10^4}$$

$$\approx \frac{1}{2} \times 10^{-4}$$

Then

$$M_{leaves} \simeq 500\,mol \times \frac{.03\,kg}{1\,mol} \simeq \boxed{15\,kg \simeq M_{leaves}}$$

accept
10 or 20 kg
(1 sig. fig.)

c) Thermal speed: accept $\bar{V}$ or V_{rms}

$$\boxed{\bar{V} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}}\ , \quad \boxed{V_{rms} = \sqrt{\frac{3kT}{m}}}\ , \quad m \simeq m_{N_2}/N_A$$

$$kT \simeq 10^{-23}\cdot 288 = 2.88\times10^{-21}\,J \qquad \simeq \frac{0.03\,kg}{1\,mol}\times\frac{1\,mol}{6\times10^{23}}$$

$$\simeq \frac{3\times10^{-2}}{6\times10^{23}} \simeq \frac{1}{2}\times10^{-25}$$

$$\boxed{m \simeq 5\times10^{-26}\,kg}$$

so

$$\bar{V} \simeq \left(\frac{8\cdot 2.88\times10^{-21}\,K}{16\times10^{-26}\,kg}\right)^{1/2}$$

$$\simeq \left(\frac{3}{2}\times10^{5}\right)^{1/2} \simeq \sqrt{\frac{30}{2}}\times10^{2} \simeq \boxed{400\,m/s \simeq \bar{V} \simeq V_{rms}}$$

$$l_m = \frac{1}{4\pi\sqrt{2}\,r^2\left(\frac{N}{V}\right)} = \boxed{\frac{kT}{4\pi\sqrt{2}\,r^2 P} = l_m} \qquad \frac{N}{V} = \frac{P}{kT}$$

$r = 4\times10^{-10}\,m$ so

$$l_m \simeq \frac{2.88\times10^{-21}}{4\pi\sqrt{2}\,(16\times10^{-20})\,10^{5}} \approx \frac{3\times10^{-21}}{12\times1.5\times16\times10^{-15}}$$

$$\simeq \frac{10^{-6}}{4\times1.5\times16} \simeq \frac{2}{64\times3}\times10^{-6}$$

$$\simeq \frac{2}{200}\times10^{-6} = \boxed{10^{-8}\,m \simeq l_m}$$

The typical separation is

$$\left(\frac{V}{N}\right)^{1/3} \simeq \left(\frac{2.88\times10^{-21}\,J}{10^{5}\ N/m^2}\right)^{1/3} \approx \left(3\times10^{-26}\right)^{1/3} = 3^{1/3}\times10^{-26/3}$$

$$\boxed{\left(\frac{V}{N}\right)^{1/3} \sim 10^{-9}}$$

Thus $\boxed{\dfrac{l_m}{\left(\frac{V}{N}\right)^{1/3}} \simeq 10}$

Rubric

1) 20 points total

a) 4 points:
- 2 points for correct algebraic form of V_{room}
- $\frac{1}{2}$ for $M_{air} = \rho_{Air} V_{room}$
- $\frac{1}{2}$ for correct numerical estimate

b) 8 points:

- 2 point for $M_{leaves} = \Delta n \times M_{air}$

  Δn: change in # of moles in room; M_{air}: molar mass of air

- 3 points for
$$\Delta n = \frac{P V_{room}}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)$$

- 2 point for recognizing P, V are constant
- $\frac{1}{2}$ point for estimating Δn numerically
- $\frac{1}{2}$ point for estimating M_{leaves} numerically

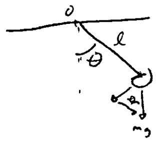
c) 8 points:
- 2 points for $\bar{v}$ or V_{rms} formula
- 1 point for estimating molar mass or particle mass correctly
- $\frac{1}{2}$ point for some numerical simplifying/algebra plug-in
- $\frac{1}{2}$ for numerical estimate of $\bar{v}$ or V_{rms}

cont'd...

- 1 point for plugging in values into l_m equation
- 1 point for full simplification within an order of magnitude to the solution.
- 1 point for estimating the typical separation $\left(\frac{V}{N}\right)^{1/3}$
- 1 point for comparing the l_m with $\left(\frac{V}{N}\right)^{1/3}$.

Bale Midterm Solution.

2)



$$\tau_o = -\ell mg\sin\theta = m\ell^2\ddot{\theta} \rightarrow \ddot{\theta} + \frac{g}{\ell}\theta = 0 \rightarrow \omega = \sqrt{\frac{g}{\ell}} \text{, or } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$L(T) = L_0(1+\alpha\Delta T)$$

$$\therefore T_0 = 2\pi\sqrt{\frac{L_0}{g}}, \quad T_f = 2\pi\sqrt{\frac{L(T)}{g}} = 2\pi\sqrt{\frac{L_0(1+\alpha\Delta T)}{g}} = T_0\sqrt{1+\alpha\Delta T} \rightarrow \frac{T_0}{T_f} = \frac{1}{\sqrt{1+\alpha\Delta T}}$$

After $T_0 = 1$ sec has passed, the new clock only reads $\left(\frac{T_0}{T_f}\right)$ seconds, so it is behind by $1-\frac{T_0}{T_f}$ seconds for every second on the original clock.

$\therefore$ Total time the new clock is behind original clock is: $\Delta t_{behind} = \left(1-\frac{T_0}{T_f}\right)t_{tot}$ time acc. to original clock.

$$\rightarrow \Delta t_{behind} = \left(1-\frac{1}{\sqrt{1+\alpha\Delta T}}\right)t_{tot} = \left(1-\frac{1}{\sqrt{1+\alpha\Delta T}}\right)(1\text{ yr})$$

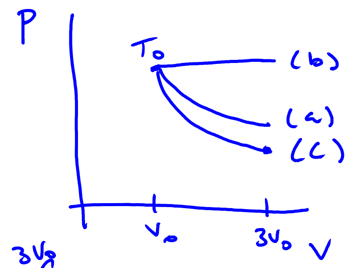
$$\approx \left[1-\left(1-\tfrac{1}{2}\alpha\Delta T\right)\right](1\text{ yr})$$ for $\alpha\Delta T \ll 1$, which is valid

$$\boxed{\Delta t_{behind} = \left(\tfrac{1}{2}\alpha\Delta T\right)(1\text{ yr}) = \tfrac{1}{2}(25\times10^{-6}\text{ K}^{-1})(8\text{ K})(1\text{ yr}) = 1\times10^{-4}\text{ yr} = 52.6\text{ mins behind}}$$

Problem 3

Thursday, February 23, 2017 12:10 AM

A monatomic ideal gas of N atoms is initially at temperature T0 in volume V0. The gas is allowed to expand slowly to a final volume 3V0 in one of three different ways: (a) at constant temperature, (b) at constant pressure, and (c) adiabatically. For each of these processes, calculate the work done by the gas, the amount of energy transferred to the gas by heating, and the final temperature. Express your answer in terms of V0, T0, the ratio of specific heats γ, and the universal gas constant.



a) T constant $\rightarrow \Delta E = 0 = Q - W \longrightarrow Q = W = \int_{V_0}^{3V_0} P\,dV = NkT_0 \ln \frac{3V_0}{V_0} = NkT_0 \ln 3$

$T_f = T_0$

b) Const. pressure $Q = nC_p \Delta T = N \underbrace{\tfrac{5}{2}}_{\text{monatomic}} k \left(\frac{P_0(3V_0)}{nR} - T_0 \right) = N\frac{5}{2}kT_0(3-1)$

$\frac{P_0(3V_0)}{nR} = T_f = 3T_0$

$\Delta E_{int} = nC_v \Delta T = N \underbrace{\tfrac{3}{2}}_{\text{monatomic}} k\, 2T_0$ $\qquad W = Q - \Delta E_{int} = 2NkT_0$

c) adiabat $Q = 0$ $\qquad PV^\gamma = \text{const} \rightarrow TV^{\gamma-1} = \text{const} \rightarrow T_{final} = T_0\left(\frac{1}{3}\right)^{\gamma-1}$

$\Delta E_{int} = nC_v \Delta T = N\frac{3}{2}kT_0\left(\left(\frac{1}{3}\right)^{\gamma-1} - 1\right)$ $\qquad W = Q - \Delta E_{int} = -\Delta E_{int}$

<u>Rubric</u>

a – 6: 2 identify $\Delta E = 0$; 2 $Q = W$; 2 W correct

b – 7: 1 T_f; 3 Q; 3 W

c – 7: 2 T_f; 2 Q; 3 W

(mistakes won't be double penalized eg in C if ΔE_{int} is wrong but they write $W = Q - \Delta E = -\Delta E$ this will be +2)

(if neither Q, W are correctly calculated then dof = 3 expressed is +2)

Vijay Narayan

4. Solution

$$\frac{\Delta Q}{\Delta t} = S \epsilon A \cos\theta$$

emissivity — ϵ; angle between sun rays and normal to surface — θ; area — A

ΔQ to melt ice given by $\Delta Q = m_{ice} L$ only

$m_{ice} = \rho_{ice}(A\delta)$ (not changing temp, only phase)

$\Rightarrow \Delta Q = \rho_{ice} A \delta L$

Solving for Δt (time needed to completely melt ice)

$$\Rightarrow \Delta t = \frac{\Delta Q}{S \epsilon A \cos\theta} = \frac{\rho_{ice}\, \delta L}{S \epsilon \cos\theta}$$

Plugging in given values of

$\rho_{ice} = 900\ kg/m^3$, $A = 100\ m^2$, $\delta = 0.05\ cm$, $\theta = 30°$, $L = 333\ kJ/kg$, $S = 1000\ W/m^2$, $\epsilon = 0.05$

$\Rightarrow \Delta t \approx 3 \times 10^4\ s$ ☒

Rubric

Correct Heating due to radiation formula (no need to derive) – 7 points

ΔQ due to Latent heat only (phase change) – 5 points

Solving for Δt symbolically – 3 points

Correct angle in formula – 1 point

Correct mass of ice symbolically – 2 point

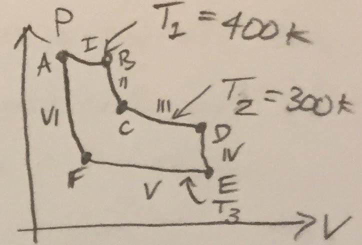
Correct Numerical value for Δt – 2 point

PHYS 7B, Section 2 (Bale) Robert McGehee

Midterm #1 2/21/17

5) a) $\Delta S_{sys} = 0$ because entropy is a state variable and the system returns to its original state after a complete cycle.

b) Labeling points on the cycle for reference:



We find T_3 by calculating the work along each path and using that the total work, W, is $W = 100\,J$.

First, we calculate the works for the adiabatic paths, Since $Q=0$, $W = -\Delta E$.

$$W_{II} = -\Delta E_{II} = -\frac{f}{2}Nk(T_2 - T_1) \quad ,$$

$$W_{IV} = -\Delta E_{IV} = -\frac{f}{2}Nk(T_3 - T_2) \quad , \qquad \Longrightarrow W_{II} + W_{IV} + W_{VI} = 0$$

$$W_{VI} = -\Delta E_{VI} = -\frac{f}{2}Nk(T_1 - T_3)$$

The adiabatic paths contribute no net work.

Next, we calculate the works for the isothermal paths. $\Delta T = 0 \Rightarrow \Delta E = 0$.

$\Rightarrow W = Q$ along the isotherms, We are told the input heats for I and III:

$$W_I = 200\,J \quad ,$$

$$W_{III} = 300\,J$$

For V, $W_V = \int P\,dV = \int_{V_E}^{V_F} \frac{NkT_3}{V}\,dV \overset{T \text{ is constant}}{=} NkT_3 \int_{V_E}^{V_F} \frac{dV}{V} = NkT_3 \ln\left(\frac{V_F}{V_E}\right)$.

Along adiabats, $PV^{\gamma} = \text{constant} \Rightarrow TV^{\gamma-1} = \text{constant}$

$$\Rightarrow T_3 V_F^{\gamma-1} = T_1 V_A^{\gamma-1}, \quad T_3 V_E^{\gamma-1} = T_2 V_D^{\gamma-1}, \quad T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$$

$$\Rightarrow \left(\frac{V_F}{V_E}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)\left(\frac{V_A}{V_D}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)\left(\frac{V_A}{V_B}\cdot\frac{V_B}{V_D}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)\cdot\left(\frac{V_A}{V_B}\cdot\frac{V_C}{V_D}\right)^{\gamma-1}\cdot\left(\frac{T_2}{T_1}\right) = \left(\frac{V_A}{V_B}\cdot\frac{V_C}{V_D}\right)^{\gamma-1}$$

$$\Rightarrow \frac{V_F}{V_E} = \frac{V_A}{V_B}\cdot\frac{V_C}{V_D}$$

We also realize that $W_I = NkT_1 \ln\left(\frac{V_B}{V_A}\right)$, $W_{III} = NkT_2 \ln\left(\frac{V_D}{V_C}\right)$

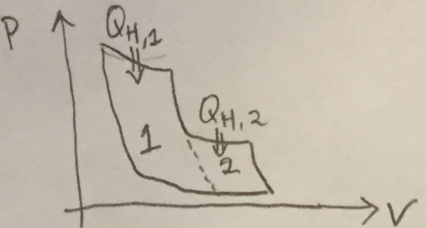
$$\text{so: } W_V = NkT_3 \ln\left(\frac{V_A}{V_B}\cdot\frac{V_C}{V_D}\right) = NkT_3\left[\ln\left(\frac{V_A}{V_B}\right) + \ln\left(\frac{V_C}{V_D}\right)\right] = -\left(\frac{T_3}{T_1}\right)W_I - \left(\frac{T_3}{T_2}\right)W_{III}$$

Adding all of the works together, we find:

$$W = W_{I} + W_{III} - \frac{W_{I}}{T_1}T_3 - \frac{W_{III}}{T_2}T_3 \Rightarrow T_3\left(\frac{W_I}{T_1} + \frac{W_{III}}{T_2}\right) = W_I + W_{III} - W$$

$$T_3 = \frac{W_I + W_{III} - W}{\frac{W_I}{T_1} + \frac{W_{III}}{T_2}}$$ Plugging in the values given $T_3 = \frac{400\,J}{\frac{1}{2}J/K + 1\,J/K} = 267\,K$

We can also find T_3 by realizing this is 2 Carnot cycles combined:



"1" subscripts refer to the first Carnot cycle and "2" to the second. From the information in the problem,

$T_{H,1} = 400\,K \quad Q_{H,1} = 200\,J$

$T_{H,2} = 300\,K \quad Q_{H,2} = 300\,J$

Both Carnot cycles have $T_L = T_3$. Finding the total work:

$$W = W_1 + W_2 = Q_{H,1} \cdot e_1 + Q_{H,2} \cdot e_2$$

For a Carnot cycle, $e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$.

$$\Rightarrow W = Q_{H,1} \cdot \left(1 - \frac{T_3}{T_{H,1}}\right) + Q_{H,2} \cdot \left(1 - \frac{T_3}{T_{H,2}}\right)$$

$$\Rightarrow T_3\left(\frac{Q_{H,1}}{T_{H,1}} + \frac{Q_{H,2}}{T_{H,2}}\right) = Q_{H,1} + Q_{H,2} - W \Rightarrow T_3 = \frac{Q_{H,1} + Q_{H,2} - W}{\frac{Q_{H,1}}{T_1} + \frac{Q_{H,2}}{T_2}}$$

Plugging in the values we again find $T_3 = 267\,K$.

We can also find T_3 by using $\Delta S = 0$: where $Q_{out} > 0$ is the heat lost during V

$$\Delta S_I = \frac{Q_{H,1}}{T_1}, \quad \Delta S_{III} = \frac{Q_{H,2}}{T_2}, \quad \Delta S_V = \frac{-Q_{out}}{T_3} = \frac{-(Q_{H,1} + Q_{H,2} - W)}{T_3}$$ (since isotherms)

$\Delta S_{II} = \Delta S_{IV} = \Delta S_{VI} = 0$ (since adiabats)

$$\Delta S = 0 \Rightarrow \left(\frac{T_3}{T_1}\right)Q_{H,1} + \left(\frac{T_3}{T_2}\right)Q_{H,2} = Q_{H,1} + Q_{H,2} - W$$

$$T_3 = \frac{Q_{H,1} + Q_{H,2} - W}{\frac{Q_{H,1}}{T_1} + \frac{Q_{H,2}}{T_2}} \Rightarrow T_3 = 267\,K$$