Physics 7A (Sec. 2) Final Exam December 12, 2002

You may use one (1) sheet of paper (both sides) not larger than 8.5" x 11", but no other papers, and no books. Exam totals 400 points

(15) (1) An unstable atomic nucleus is at rest, and decays by emitting an electron and a neutrino. (The neutrino is a "fundamental" particle, as is the electron.) The directions of the emitted electron and neutrino are perpendicular to each other. The measured linear momentum of the emitted electron is 1.22×10^{-22} kg m sec^{-1}, and that of the neutrino is 6.4×10^{-23} kg m sec^{-1}. (a) Calculate the magnitude of the linear momentum of the recoiling nucleus; (b) Calculate the angle θ made by the direction of the recoiling nucleus with the direction of the emitted neutrino. [Part (a) = 10 points, (b) = 5 points]

(15) (2) A vertical spring is suspended at one end and a mass of 0.40 kg is attached to the other end. When the mass is attached, the spring stretches by 0.20 meter. The spring is then stretched vertically downward by an additional 0.10 meter and released. Write the equation giving the displacement $y(t)$ of the spring as a function of time, assuming simple harmonic motion.

(30) (3) Two sinusoidal transverse traveling waves, each with an amplitude of 2.0 meters, move in the $(+x)$ direction the same a speed of 5.0 m/sec. Both have a wavelength of (2π) meters, but they differ in phase by $(\pi/4)$ radians. (a) Write down the equation of the resultant wave produced by the superposition of these two waves; (b) Calculate the displacement of the resultant wave at $x = 1.0$ meter when the time $t = 0.2$ sec.; (c) Write down the equation of a wave

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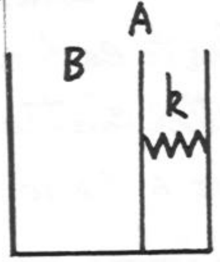
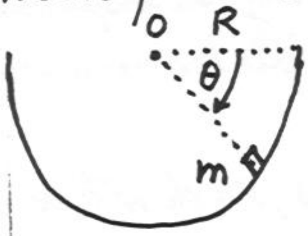
(3) [continued] (of the same phase, amplitude, etc) as your answer to (a), which, when superposed with your answer to (a), will produce a standing wave; (d) Calculate the distance (in meters) between the nodes of your answer to (c). [(a) = (c) = 10; (b) = (d) = 5]

(30) (4) An unusual water container is shown in the drawing. Wall A



can move, and, when moved to the right, compresses the spring of force constant k. The dimension of the container in the direction normal to the plane of the paper is l. The region B of the vessel is filled to a final depth h with a fluid of mass density ρ kg m^{-3}. The wall A then moves a distance d to the right and stops. The external atmospheric pressure on the fluid is p_0 (in Pa). Calculate the value of the distance d (in terms of the quantities given in the question).

(30) (5) A small body of mass m starts from rest at the top of a hemispherical bowl of radius R, as shown. The body slides



down the frictionless inner surface of the bowl, always remaining in contact with the surface. (a) Calculate the angular acceleration vector $\underline{\alpha}$ of the motion of mass m. (b) Sketch the magnitude $|\underline{\alpha}|$ as a function of the angle θ for $0 \leq \theta \leq (\pi/2)$; (c) Describe in words the variation of $|\underline{\alpha}|$ with θ shown in your answer to (b). [Part (a) = 20, (b) = 5, (c) = 5 points]

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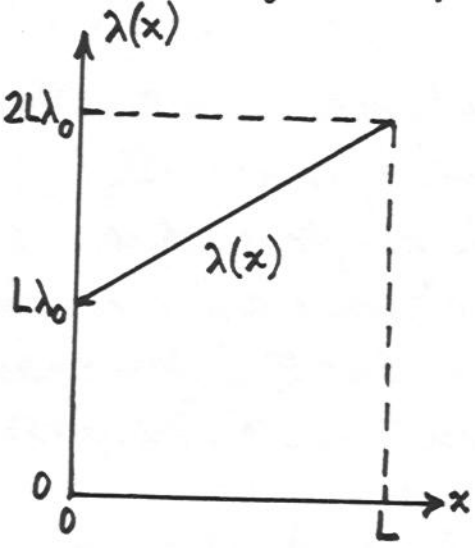
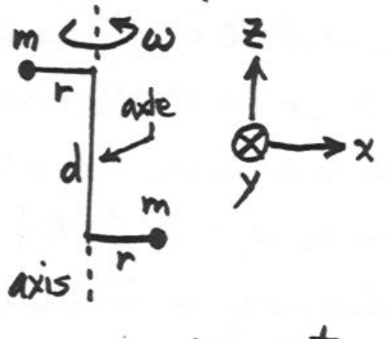
(30)(6) A rope of mass M and length L (with a uniform linear mass density in kg m^{-1}) hangs vertically from a tree limb. Calculate the tension $T(x)$ in the rope at a distance x from the bottom of the rope.

(40)(7) Given a long thin rod of length L and mass M. The linear mass density $\lambda(x)$ of the rod (in kg m^{-1}) is as shown in the graph.



On the graph, x is the distance from one end of the rod, λ_0 is a constant (with units kg m^{-2}) and the plot of $\lambda(x)$ as a function of x is linear. (a) Calculate the moment of inertia of the rod about an axis normal to the end of the rod at the end of the rod at which $\lambda = L\lambda_0$. Your answer will be in terms of M and L; (b) Calculate the distance x_{cm} of the center of mass of the rod from the end of the rod at which $\lambda = L\lambda_0$. Your answer will be in terms of L. [(a) = (b) = 20 pts each]

(40)(8) Two massless rods of length r are mounted perpendicular to an axle of length d; each rod ends in a point mass m. The



axle rotates as shown with an angular speed of ω radians/sec. Assume that the axle and the axis of rotation lie along the z-direction, and that the y- and x-directions are as shown. Calculate the total angular momentum vector $\underline{L}$ of the rotating system.

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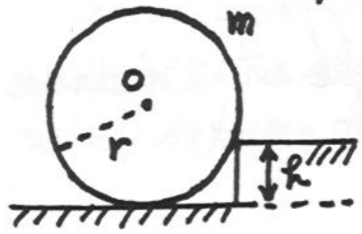
(40) (9) Given a uniform circular wheel (a disc) in front of a step of height h, as shown. The mass of the wheel is m and its radius is r. A horizontal force $\underline{F}$ is applied at the center O of the wheel. Calculate the value of $|\underline{F}|$ for which the wheel will just lift. Assume that, at the instant the wheel lifts, the wheel is in rotational equilibrium.



(40) (10) A force $\underline{F} = F_0 x^2 y^2 (\hat{i} + \hat{j})$, where F_0 is a constant, moves a particle of mass m from the point of coordinates $(0, -a)$ to the point $(a, 0)$ along a path in the xy-plane which is the shorter arc of the curve $x^2 + y^2 = a^2$, where a is a constant. (a) Calculate the work W done by the force $\underline{F}$; (b) Show explicitly that your units are correct in your answer to part (a). [Part (a) = 35, (b) = 5]

(45) (11) Given a particle of mass m which is subject only to a frictional force $\underline{f}$. The displacement of the particle is $x(t)$, its speed is $v(t)$, and the force $\underline{f}$ has a magnitude f which is directly proportional to v, with constant of proportionality b; $\underline{f}$ is antiparallel to the velocity vector $\underline{v}$. (a) Write down the equation of motion of the particle with $v(t)$ as the dependent variable and time t as the independent variable; (b) If the initial speed of the particle is v_0, solve the equation in part (a) for the function $v(t)$; (c) Calculate the characteristic time τ for the speed of the particle to decrease to $(1/e)v_0$; (d) Make a sketch showing the variation of $v(t)$ with time over a time interval t, where $0 \leq t \leq 2\tau$, indicating on the graph the values of $v(t)$ at the times $t = \tau$ and $t = 2\tau$; (e) Derive an

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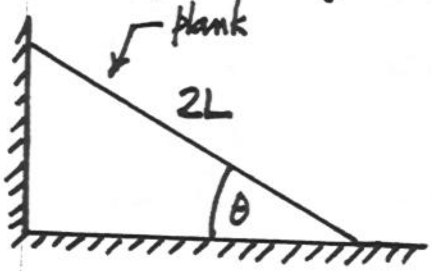
(11) [continued] expression for the kinetic energy $K(t)$ of the particle as a function of time t; (f) Calculate the characteristic time τ' for the kinetic energy to decay to $(1/e)$ of its initial value; (g) Express τ' in terms of τ, and suggest (in words) a physical reason for the numerical relationship between τ' and τ. [Parts (a) = (b) = 10 points each; (c) = (d) = (e) = (f) = (g) = 5 pts each]

(45) (12) A plank of mass m and length $(2L)$ leans against a wall, making an angle θ with the horizontal. There is no friction between the ends of the plank and the wall and the floor.



(a) Write down Newton's Second Law for the horizontal motion of the center of mass of the plank as the plank slips down without friction. Your equation expressing the Second Law must be in terms of the angle θ and its time derivatives (as well as constants); (b) Use your result in part (a) to determine the condition which must hold when the top of the plank loses contact with the wall. This condition is to be expressed in terms of θ and its time derivatives. [Part (a) = 35 points, (b) = 10 points]