Solutions to Midterm 1

Problem 1. (a) Car B is in rectilinear motion. Attach a translating (x, y) frame to B with the x-axis at 30° to $\mathbf{v}_B$.

$$v_A = 30\frac{5280}{60^2} = 30\frac{44}{30} = 44 \text{ ft/sec} \qquad \text{along } \mathbf{j}$$
$$a_A = \frac{v_A^2}{\rho} = \frac{44^2}{500} = 3.872 \text{ ft/sec}^2 \qquad \text{along } \mathbf{i}$$
$$a_B = 5\frac{44}{30} = 7.333 \text{ ft/sec}^2 \qquad \text{along } -\mathbf{v}_B$$

Observe that

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$
$$\Rightarrow \quad 3.872\mathbf{i} = -7.333\cos 30°\,\mathbf{i} - 7.333\sin 30°\,\mathbf{j} + \mathbf{a}_{A/B}$$
$$\Rightarrow \quad \mathbf{a}_{A/B} = 10.223\mathbf{i} + 3.667\mathbf{j}$$
$$\Rightarrow \quad a_{A/B} = 10.86 \text{ ft/sec}^2$$

The solution may also be obtained graphically from a vector diagram of accelerations.
(b) An (x, y) frame attached to car A (with the y-axis in direction of $\mathbf{v}_A$) is a rotating system. The acceleration $\mathbf{a}_{\text{rel}}$ of car B as observed from car A is such that $\mathbf{a}_{\text{rel}} \neq -\mathbf{a}_{A/B}$.

Problem 2. Blocks A and B are in rectilinear motion. All positions are measured from a vertical line through the centers of pulleys.

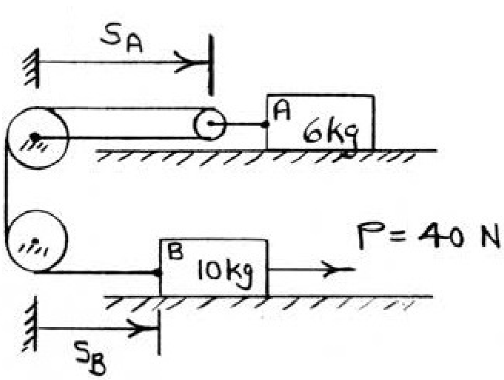
$$2s_A + s_B = C$$
$$\Rightarrow \quad 2v_A + v_B = 0$$

If A moves by 0.4 m, B would move by a distance of $\Delta s_B = 0.8$ m. The only force that performs work is the external force P. The work of tension is zero because tension in a string occurs as equal and opposite internal forces with the same displacement. When the system moves from an initial rest configuration to a final configuration after A has moved 0.4 m,

$$U_{1-2} = \Delta T = T_2$$
$$\Rightarrow \quad P\Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B(-2v_A)^2$$
$$\Rightarrow \quad v_A = \sqrt{\frac{40(0.8)}{3+5(4)}} = -1.18 \text{ m/s}$$

In addition,

$$v_B = -2v_A = 2.36 \text{ m/s}$$



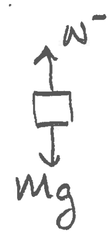
3)

a) $m_p \underline{v}_p + m_b \underline{v}_b = m_p \underline{v}_p' + m_b \underline{v}_b' \qquad (v_p' = v_b' = v')$

$\Rightarrow v' = \dfrac{m}{m+M} v = \dfrac{0.05}{(0.05+25)}(800) = \underline{\underline{1.5968 \text{ m/s}}}$

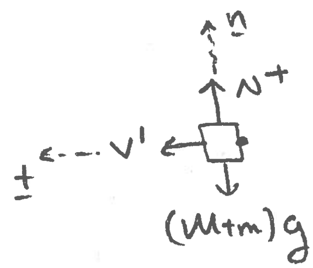
b)

Before



$N^- = 245.25$ N

After



Apply $\underline{F} = m\underline{a}$

$\underline{n}: \; N^+ - (M+m)g = (M+m)\dfrac{v'^2}{\ell}$

$N^+ = (25.05)\,9.81 + (25.05)\dfrac{1.5968^2}{2}$

$= 277.6764$ N

$\Rightarrow [\![N]\!] = N^+ - N^- = \underline{\underline{32.4264 \text{ N}}}$ Jump in tension

c) $T_1 + V_1 + \cancel{W_{12}}^{\,0} = T_2 + V_2$ Datum → Equilibrium

$\frac{1}{2}(M+m)\,v'^2 = (m+M)\,g\ell\,(1-\cos\theta)$

$\frac{1}{2}(25.05)\,1.5968^2 = (25.05)(9.81)(2)\,(1-\cos\theta)$

$\cos\theta = 0.935 \quad \Rightarrow \theta = \underline{\underline{20.7684^\circ}}$