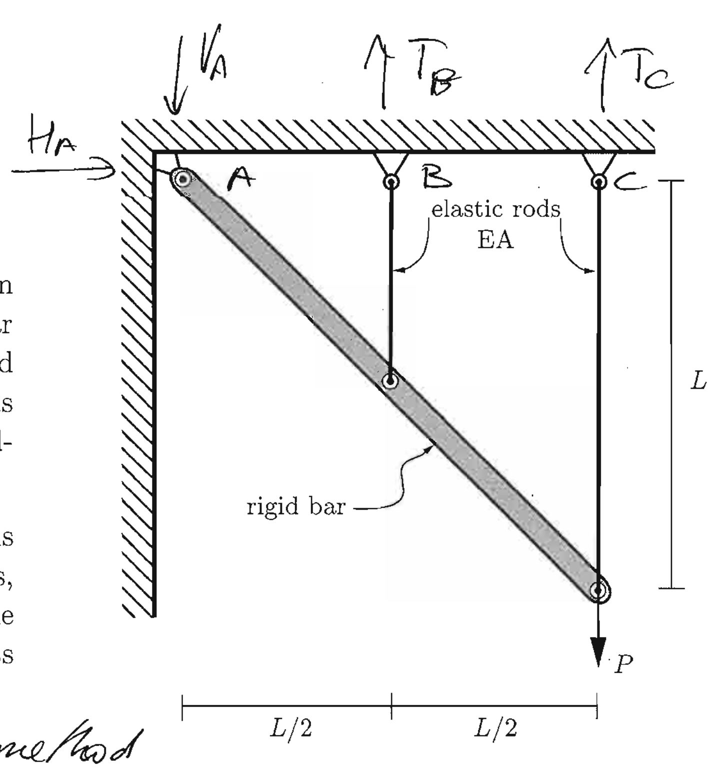
Problem #1 (40%)

A rigid bar hangs from two rods as shown in the figure. The rods can be considered linear elastic with an equal Young modulus E and have a cross section area A. All connections are pinned, and all members can be considered weightless.

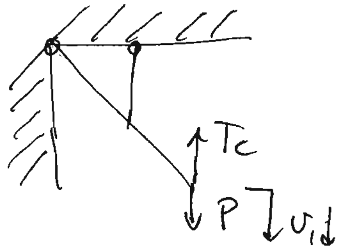
If the vertical load P shown in the figure is applied, determine (1) the force in the rods, (2) the reactions at the top left end of the bar, and (3) the deflection of the bar at its bottom right tip.



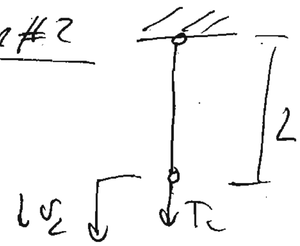
Statically indeterminate ⇒ Force method (3 steps)
(degree of indeterminancy)

$\boxed{\text{STEP 1}}$ Release the system, e.g. disconnect the right rod (leaving the force)

Problem #1



Problem #2



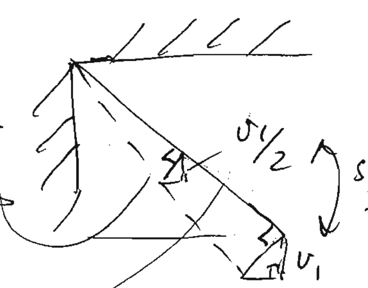
flexibility of right rod

$\downarrow v_2 = \frac{T_c L}{EA}$

$\boxed{\text{STEP 2}}$ Solve for v_1 and v_2 (in terms of P and T_c)

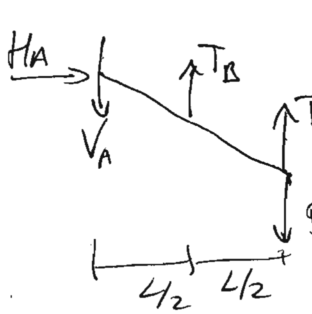
Problem #1 Kinematics

rigid bar ⇒ moves perpendicular to the bar



similar triangles

Statics



$\sum M = 0 \Rightarrow T_B \frac{L}{2} = (P - T_c) L$

$\Rightarrow T_B = 2(P - T_c)$

$(\sum F = 0) \Rightarrow H_A = 0$

$V_A = T_B + T_c - P = P - T_c$

$\frac{v_1}{2}$ = stretch of the middle rod

$= \frac{T_B (L/2)}{EA}$ length of middle rod

$\downarrow v_1 = 2\frac{T_B L/2}{EA} = \frac{2(P - T_c) L}{EA}$

$\boxed{\text{STEP 3}}$ Impose back compatibility

$$\downarrow v_1 = \downarrow v_2$$

$$\frac{2(P-T_C)L}{EA} = \frac{T_C L}{EA} \Rightarrow 2(P-T_C) = T_C$$

$$\rightarrow \boxed{T_C = \frac{2}{3}P}$$

From Step 2, $T_B = 2(P-T_C) = \frac{2}{3}P$

$V_A = P - T_C = \frac{1}{3}P$

$H_A = 0$

$$\Rightarrow \boxed{\begin{aligned} T_C &= \frac{2}{3}P \\ T_B &= \frac{2}{3}P \\ V_A &= \frac{1}{3}P \\ H_A &= 0 \end{aligned}}$$

(all positive $\Rightarrow$ directions as originally assumed)

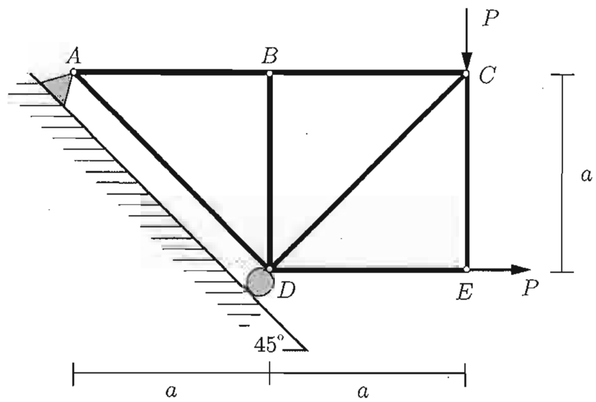
Deflection at the tip: $\boxed{\downarrow v_1 = \downarrow v_2 = \frac{T_B L}{EA} = \frac{2}{3}\frac{PL}{EA}}$

from Step 2

Problem #2 (25%)

1. Determine the forces in all the members in the truss of the figure when the two loads shown are applied together (horizontal and vertical, respectively, each of value P). Indicate clearly if the member is in tension or compression.

2. If all the members have the same $0.1 \times 0.1\ m^2$ square cross section, determine the maximum load value P that can be applied with a factor of safety of 1.5 if the material can only take $10 MPa$ in tension or compression.



All angles are 45° or 90°

Remark: Express your results in terms of the length a if needed.

① Zero-force members $\boxed{F_{BD} = F_{EC} = 0}$

Joint E: $F_{EC} = 0$, F_{ED}, P $\Rightarrow \boxed{F_{ED} = P}$

Joint C: F_{BC}, P, F_{CD}, $F_{EC} = 0$

compression

$F_{CD} \cos 45 + P = 0 \Rightarrow \boxed{F_{CD} = -\sqrt{2} P}$

Joint B: F_{AB}, $F_{BC} = P$, $F_{BD} = 0$ $\Rightarrow \boxed{F_{AB} = P}$

$F_{BC} + F_{CD} \sin 45 = 0 \rightarrow \boxed{F_{BC} = +P}$

Joint D: F_{AD}, 0, $F_{CD} = -\sqrt{2} P$, $F_{ED} = P$, R_D

$\rightarrow F_{AD} - F_{ED} \cos 45 = 0 \Rightarrow \boxed{F_{AD} = \frac{\sqrt{2}}{2} P}$

All tension except member CD

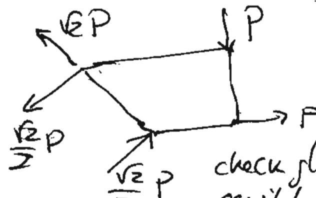
Remark: We are not asked for the reactions, but we can get them easily as

$\rightarrow R_D + F_{CD} + F_{ED} \sin 45° = 0 \Rightarrow R_D = -F_{CD} - F_{ED} \frac{\sqrt{2}}{2} = \sqrt{2} P - P\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} P$

Joint A: R_{A_2}, $F_{AB} = P$, R_{A_1}, $F_{AD} = \frac{\sqrt{2}}{2} P$

$\Rightarrow R_{A_1} = -F_{AB} \cos 45° = -\frac{\sqrt{2}}{2} P$

$R_{A_2} = F_{AD} + F_{AB} \sin 45 = \sqrt{2} P$



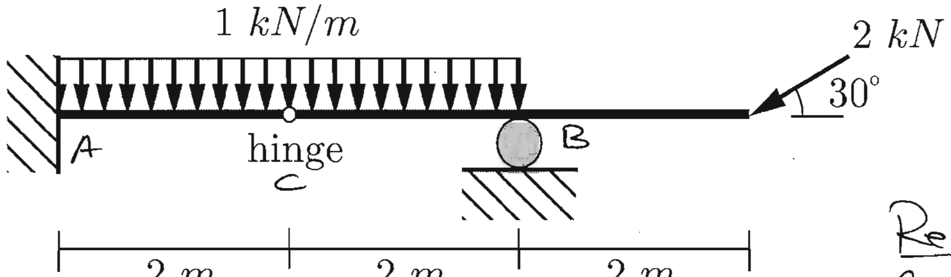
check global equilibrium ✓

② Maximum force in all members $= \sqrt{2} P$ (compression)

$$\Rightarrow \frac{\sqrt{2} P}{\underset{10^{-2} m^2}{A}} \leq \frac{\overset{= 10 MPa}{\sigma_{max}}}{\underset{1.5}{F.S.}} \Longrightarrow \boxed{P_{max} = 47.14\ kN}$$

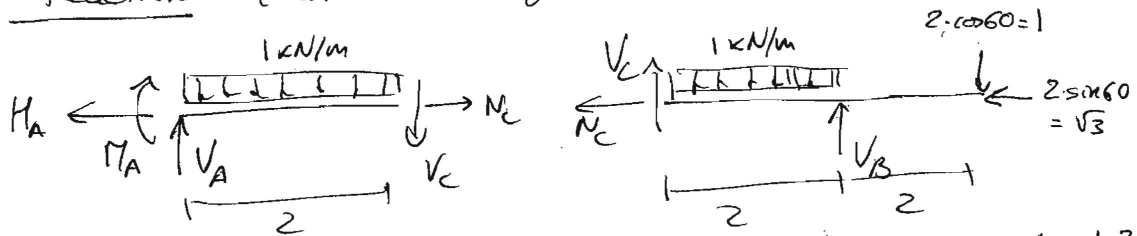
Problem #3 (35%)

Draw the axial force, transversal shear force and bending moment diagrams for the beam shown in the figure. Indicate the characteristic values (min/max values, values at the ends and supports, slopes, linear/parabolic/cubic distributions,...).



Remark: The shear force at the hinge happens to be 0 (by chance, for the given values)

Reactions: (cut at the hinge, no moment)



$\Rightarrow N_C = -\sqrt{3}$

$V_B \cdot 2 = 1\cdot 4 + 1\cdot 2\cdot 1 \Rightarrow \underline{V_B = 3}$

$V_C + V_B = 1 + 1\cdot 2 \Rightarrow \underline{V_C = 0}$

$\rightarrow H_A = N_C = -\sqrt{3}$

$V_A - V_C - 1\cdot 2 \Rightarrow \underline{V_A = 2}$

$M_A + \underset{0}{V_C}\cdot 2 + 1\cdot 2\cdot 1 = 0 \Rightarrow M_A = -2$ opposite than assumed

Diagrams