Midterm Exam

Math 53, March 2, 2017. Instructor: E. Frenkel

You have to solve the following 10 equally-weighted problems.

You must write your name and SID number at the top of every page.

Your solution to each problem must be written on the corresponding page. Please encircle your final answers.

You are not allowed to use any materials during the exam except for a list of formulas *handwritten* by yourself on *one side* of a standard size sheet of paper (*no printouts, no photocopies*). No calculators are allowed during the exam.

Justify your answers. You will not get full credit for an unsubstantiated or unjustified answer.

YOUR NAME: _____

YOUR SID: _____

NAME OF YOUR GSI: _____

NAME: STUDENT ID:

Problem 1. Consider the curve in $\mathbb{R}^2$ defined by the parametric equations

$$x = \sin t, \qquad y = \cos^2 t + 2\sin t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

Find a Cartesian equation for this curve, sketch the curve, and indicate with an arrow the direction in which the curve is traced as the parameter t is increasing.

NAME: STUDENT ID:

Problem 2. Sketch the curve $r = 4\sin\theta$ and find the area inside this curve but outside the circle $r = 2$.

NAME: STUDENT ID:

Problem 3. Find the point at which the line through the point $(3, 2, 1)$ and normal to the plane $2x - y + 2z = -3$ meets this plane.

NAME: STUDENT ID:

Problem 4. Find an equation of the plane containing the point $(1, 2, 3)$ and the line given by the equations

$$\frac{x-1}{3} = \frac{y-3}{2} = z - 5.$$

NAME: STUDENT ID:

Problem 5. Consider the space curve defined by the parametric equations

$$x = t^4 + t, \qquad y = t^3 - 2t, \qquad z = \frac{1}{1+t}.$$

Find an equation of the normal plane to this curve at the point corresponding to $t = 1$.

NAME: STUDENT ID:

Problem 6. Identify by name the surface given by the equation

$$x^2 + 4x - y^2 + 2y + z^2 + 2z = 0$$

and sketch this surface together with the appropriately labeled coordinate axes.

NAME: STUDENT ID:

Problem 7. Use differentials to estimate the volume of tin used in a closed tin can of height 20 cm and diameter 10 cm if the top lid has thickness 0.2 cm, the bottom lid has thickness 0.3 cm, and the side thickness of the tin is 0.1 cm. (Give the answer as a multiple of π.)

NAME: STUDENT ID:

Problem 8. Write down an equation of the tangent plane to the surface

$$-5xz^3 + y^2 - 3x^2z = 1$$

at the point $(1, 3, 1)$.

NAME: STUDENT ID:

Problem 9. Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x \sin^3 y}{x^2 + y^6}$$

does not exist.

NAME: STUDENT ID:

Problem 10. Show that if $z = f(x, y)$ is a differentiable function, and $x = e^s, y = e^t$, then

$$\frac{\partial^2 z}{\partial s^2} + \frac{\partial^2 z}{\partial t^2} = x^2 \frac{\partial^2 z}{\partial x^2} + x \frac{\partial z}{\partial x} + y^2 \frac{\partial^2 z}{\partial y^2} + y \frac{\partial z}{\partial y}.$$