Chemical Engineering 150A
Midterm Exam
Wednesday, February 22, 2017
7:10 pm – 8:00 pm

The exam is 100 points total.

Name: ______________________________ (in Uppercase)

Student ID: ___________________________

You are allowed one 8.5”×11” sheet of paper with your notes on both sides and a calculator for this exam.

The exam should have 16 pages (front and back) including the cover page.

Instructions:

1) Please write your answers in the box if provided.
2) Do your calculations in the space provided for the corresponding part. Any work done outside of specified area will not be graded.
3) Please sign below saying that you agree to the UC Berkeley honor code.
4) The exam contains two problems. Each problem has sub-parts to it.
5) Use the blank white full pages behind the question pages as scratch sheets.

Honor Code:

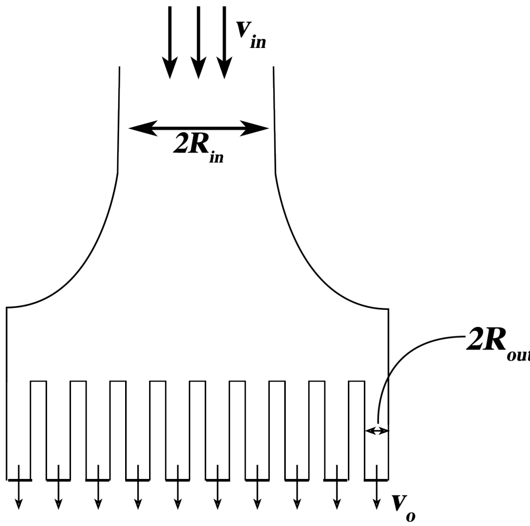
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Signature: _

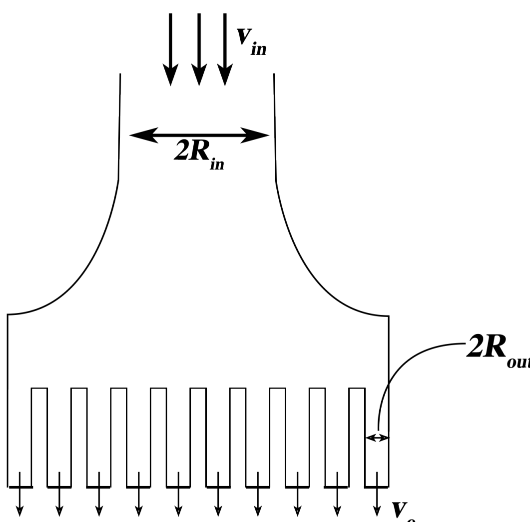
1.a	1.b	1.c	2.a	2.b	2.c	2.d	2.e	2.f	2.g	2.h	Total

Problem 1. (20 points)

Consider the showerhead depicted below. Fluid is pumped in through the main pipe, which is a cylindrical pipe with a radius of 10 cm, and exits the showerhead via 10 parallel cylindrical nozzles, each of which has a radius of 1.0 cm. The inlet flow velocity (v_{in}) and outlet flow velocity (v_o) of each outlet nozzle can be considered to be plug flow, with an inlet flow velocity of 1.0 cm/s. The outlet flow velocity is the same across all the outlet nozzles.



a) Directly on the diagram below, use a dotted line to draw the control volume that will allow you to calculate the fluid exit velocity. Is this a microscopic or macroscopic analysis? (5 points)



+3 to draw reasonable CV with inlet and outlet (many answers are correct here)
+2 to state that the analysis must be macroscopic

b) Derive an equation for the outlet velocity (v_o) of an outlet nozzle in terms of inlet velocity (v_{in}), the number of nozzles n, inlet and outlet radii R_{in} and R_{out}, and any other material parameters needed. Leave all quantities as variables and write your final answer in the box provided below. (10 points)

$$\dot{m}_{in} = \dot{m}_{out}$$

+4 points to set up mass balance correctly

$$\rho v_{in} A_{in} = \rho v_o A_o n$$

+3 points to solve for v_o

$$v_o = \frac{v_{in}}{n} \times \frac{A_{in}}{A_{out}}$$

$$v_o = \frac{v_{in} \pi R_{in}^2}{n \pi R_{out}^2}$$

+3 points for correct answer simplified in terms of v_{in}, n, R_{in}, and R_{out}. Final expression must be in terms of these variables as requested in the problem statement for full credit (no numerical values substituted)

$$v_o = \frac{v_{in}}{n}\left(\frac{R_{in}}{R_{out}}\right)^2$$

$$\boxed{v_o = \frac{v_{in}}{n}\left(\frac{R_{in}}{R_{out}}\right)^2}$$

c) If the fluid density is 2.0 g/cm^3, please calculate both the outlet velocity and outlet mass flow rate. Write your answers in terms of cm/s and g/s respectively. (5 points)

3 for correct v_o
2 for correct m_o

Outlet velocity $v_o = 10 \frac{cm}{s}$

Outlet mass flow rate $\dot{m}_o = 63 \frac{g}{s}$

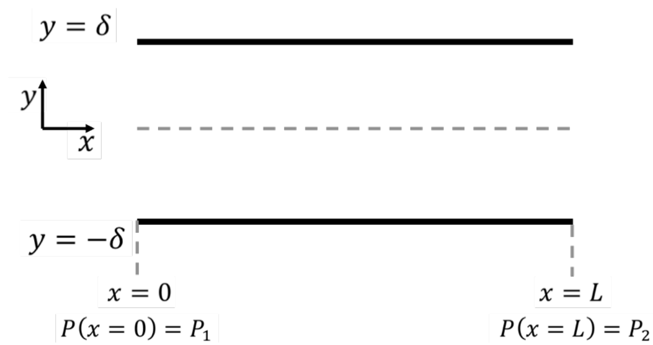
Problem 2. (80 points)

Suppose we have a fluid between two parallel flat plates separated by a distance 2δ and length L. A pressure of P_1 is applied at the inlet on the left of the two plates and a lower pressure P_2 is present at the outlet on the right.

Assume that the system is at steady state and that the velocity of the fluid has the following form:

$$\underline{v} = v_x(y)\underline{e}_x.$$

A schematic of this setup is given below along with a coordinate system.



a. Is this flow incompressible or not? Prove it. (10 points)

+4 Condition for incompressibility: $\underline{\nabla} \cdot \underline{v} = 0.$

+3 Substitute the velocity profile form: $\frac{\partial(v_x(y))}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(0)}{\partial z} = 0$

+3 Flow is incompressible.

b. Assume pressure is only a function of x and varies linearly along x. What is $P(x)$ as a function of the given variables? Write the answer in the box provided below. (5 points)

$P(x) = ax + b$

Use $P(x=0) = P_1$ and $P(x=L) = P_2$ to determine a and b.

$$a = \frac{P_2 - P_1}{L}$$

$$b = P_1$$

+5 $P(x) = \frac{P_2-P_1}{L}x + P_1$

c. What is $\underline{\nabla P}$ in the vectorial form? (5 points)

+2 $\underline{\nabla P} = \frac{\partial P}{\partial x}\underline{e_x} + \frac{\partial P}{\partial y}\underline{e_y} + \frac{\partial P}{\partial z}\underline{e_z}$

+3 $\underline{\nabla P} = \frac{P_2-P_1}{L}\underline{e_x}$

(If the explicit formula for P(x) was not substituted in this section, but it was correctly used in part e or g, full credit will be given)

d. The fluid flowing between the plates can be described by the following constitutive relationships between shear stress and velocity gradients, where μ is the coefficient of viscosity.

Please <u>circle</u> the components that are *non-zero*. (10 points)

$$\tau_{xx} = \mu \frac{\partial v_x}{\partial x} \qquad \tau_{xy} = \frac{1}{2}\mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] \; +5 \qquad \tau_{xz} = \frac{1}{2}\mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

$$\tau_{yx} = \frac{1}{2}\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \; +5 \qquad \tau_{yy} = \mu \frac{\partial v_y}{\partial y} \qquad \tau_{yz} = \frac{1}{2}\mu\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]$$

$$\tau_{zx} = \frac{1}{2}\mu\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right] \qquad \tau_{zy} = \frac{1}{2}\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right] \qquad \tau_{zz} = \mu \frac{\partial v_z}{\partial z}$$

e. Give the Cauchy momentum balance *only* in the *x*-direction and simplify it combining results from previous parts. Write the final ordinary differential equation in the box. (20 points)

+5 $$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} + \frac{\partial(\rho v_x v_z)}{\partial z} = -\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho g_x$$

+10 $$\cancel{\frac{\partial(\rho v_x)}{\partial t}} + \cancel{\frac{\partial(\rho v_x v_x)}{\partial x}} + \cancel{\frac{\partial(\rho v_x v_y)}{\partial y}} + \cancel{\frac{\partial(\rho v_x v_z)}{\partial z}} = -\frac{\partial P}{\partial x} + \cancel{\frac{\partial \tau_{xx}}{\partial x}} + \frac{\partial \tau_{yx}}{\partial y} + \cancel{\frac{\partial \tau_{zx}}{\partial z}} + \cancel{\rho g_x}$$

$$\frac{\partial P}{\partial x} = \frac{\partial \tau_{yx}}{\partial y}$$

All signs and subscripts **must** be correct to receive full credits

+5 $$\boxed{\frac{P_2 - P_1}{L} = \frac{1}{2}\mu \frac{\partial^2 v_x}{\partial y^2}}$$

f. Give appropriate boundary conditions for the flow. Write the answers in the box. (5 points)

+2.5 $v_x(y = -\delta) = 0$

+2.5 $v_x(y = \delta) = 0$

g. Solve the ordinary differential equation derived in part (e) for the velocity profile, $v_x(y)$. Write the answer in the box. (20 points)

Integrate twice. +8

$$v_x(y) = \frac{P_2 - P_1}{\mu L} y^2 + C_1 y + C_2$$

Apply boundary conditions.

$$v_x(\delta) = \frac{P_2 - P_1}{\mu L}\delta^2 + C_1\delta + C_2 = 0$$

$$v_x(-\delta) = \frac{P_2 - P_1}{\mu L}\delta^2 - C_1\delta + C_2 = 0$$

Combine:

$$\frac{P_2 - P_1}{\mu L} + C_2 = 0$$

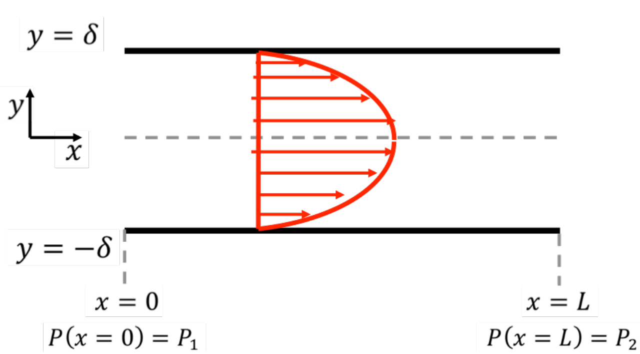
+5 $C_2 = -\frac{P_2 - P_1}{\mu L}\delta^2$

+5 $C_1 = 0$

Substitute.

+2 $\quad v_x(y) = \frac{P_2 - P_1}{\mu L}\delta^2\left[\frac{y^2}{\delta^2} - 1\right]$

h. Sketch the flow profile in the figure provided. (5 points)



+5 for parabolic flow to the right