University of California, Berkeley
Mechanical Engineering
ME 106, Fluid Mechanics
Midterm 1, Fall 2015

Last name: ____________________

First name: ____________________

Student ID: ____________________

Discussion: ____________________

Notes:

- You solution procedure should be legible and complete for full credit (use scratch paper as needed).
- You may use a calculator with simple arithmetic operations.

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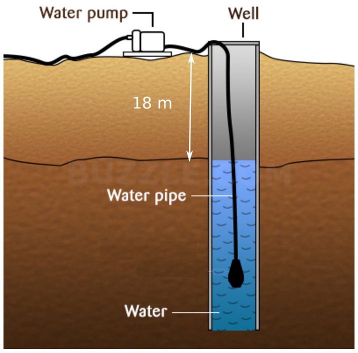
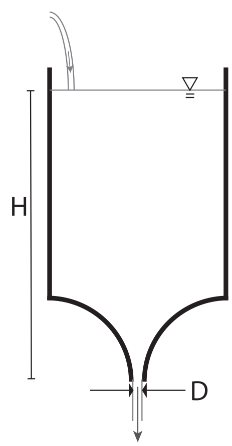
1. Because of the drought, your neighbor Ted considers drilling a well to reach the Mocho Subbasin aquifer located 18 m below ground level, and installing a pump in his backyard to bring water up to his garden. Explain why Ted's idea is bound to fail no matter how good his pump is.

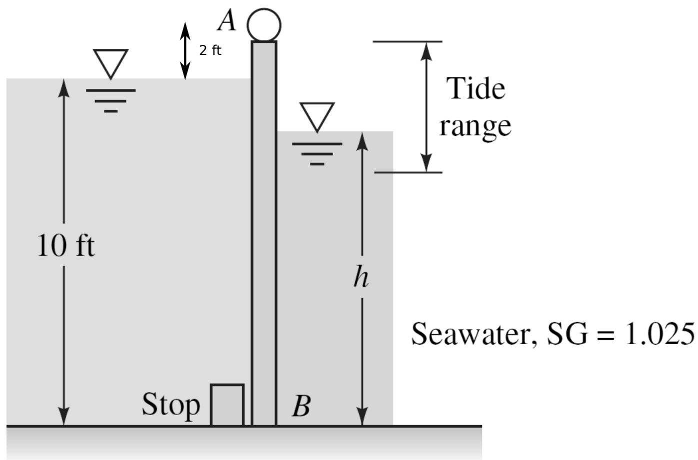


2. What does Bernoulli's equation represent, and what does each term mean? List the assumptions that we have made to arrive at Bernoulli's equation.

3. Water flows out of a cylindrical tank of height H as shown. The top surface is static (velocity is zero) since it's filled at the same rate water exits. Due to the small size of the exit, surface tension contributes to a non-negligible increase in pressure inside the fluid jet. Find the exit velocity. Express your answer in terms of the surface tension coefficient σ, specific weight of the fluid γ, jet diameter D, and fluid height H. You may assume viscosity is negligible and that the exit jet is cylindrical in shape.



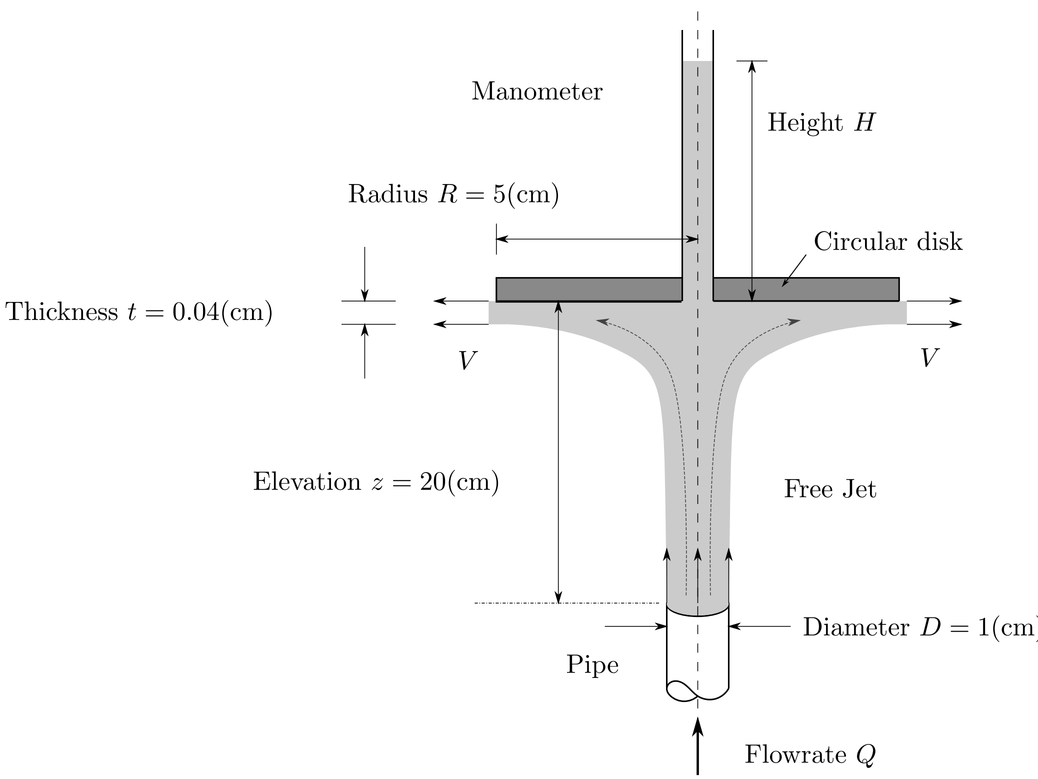
4. Gate AB is 5 ft wide into the paper and opens to let fresh water out when the ocean tide is dropping. The hinge at A is 2 ft above the freshwater level. At what ocean level h will the gate first open? Neglect the weight of the gate.



5. Water exits a pipe with flowrate Q as a free jet and strikes a circular disk of radius $R = 5$(cm) at elevation $z = 20$(cm) above the pipe. The jet exits from the sides of the disk horizontally with layer thickness $t = 4$(mm), as shown. The flow geometry is axi-symmetric. The center of disk has a hole where a manometer is installed. The water in the manometer rises up to height H and remains static. Neglect viscous effects.

   (a) Determine the flowrate Q in the pipe. (Hint: use Bernoulli and conservation of mass)

   (b) Determine the manometer reading H.



Summary of Equations:

Chapter 1:

Specific weight	$\gamma = \rho g$
Specific gravity	$SG = \frac{\rho}{\rho_{H_2O@4\,^\circ C}}$
Ideal gas law	$\rho = \frac{p}{RT}$
Newtonian fluid shear stress	$\tau = \mu \frac{du}{dy}$
Bulk modulus	$E_v = -\frac{dp}{d\forall/\forall}$
Speed of sound in an ideal gas	$c = \sqrt{kRT}$
Capillary rise in a tube	$h = \frac{2\sigma \cos\theta}{\gamma R}$

Chapter 2:

Pressure gradient in a stationary fluid	$\frac{dp}{dz} = -\gamma$
Pressure variation in a stationary incompressible fluid	$p_1 = \gamma h + p_2$
Hydrostatic force on a plane surface	$F_R = \gamma h_c A$
Location of hydrostatic force on a plane surface	$y_R = \frac{I_{xc}}{y_c A} + y_c$
	$x_R = \frac{I_{xyc}}{y_c A} + x_c$
Buoyant force	$F_B = \gamma \forall$
Pressure gradient in rigid-body motion	$-\frac{\partial p}{\partial x} = \rho a_x, \quad -\frac{\partial p}{\partial y} = \rho a_y, \quad -\frac{\partial p}{\partial z} = \gamma + \rho a_z$
Pressure gradient in rigid-body rotation	$\frac{\partial p}{\partial r} = \rho r \omega^2, \quad \frac{\partial p}{\partial \theta} = 0, \quad \frac{\partial p}{\partial z} = -\gamma$

Chapter 3:

Streamwise and normal acceleration	$a_s = V \frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{\mathscr{R}}$
Force balance along a streamline for steady inviscid flow	$\int \frac{dp}{\rho} + \frac{1}{2} V^2 + gz = C$ (along a streamline)
The Bernoulli equation	$p + \frac{1}{2}\rho V^2 + \gamma z =$ constant along streamline
Pressure gradient normal to streamline for inviscid flow in absence of gravity	$\frac{\partial p}{\partial n} = -\frac{\rho V^2}{\mathscr{R}}$
Force balance normal to a streamline for steady, inviscid, incompressible flow	$p + \rho \int \frac{V^2}{\mathscr{R}} dn + \gamma z =$ constant across the streamline
Velocity measurement for a Pitot-static tube	$V = \sqrt{2(p_3 - p_4)/\rho}$
Free jet	$V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh}$
Continuity equation	$A_1 V_1 = A_2 V_2$, or $Q_1 = Q_2$
Flow meter equation	$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$
Sluice gate equation	$Q = z_2 b \sqrt{\frac{2g(z_1 - z_2)}{1 - (z_2/z_1)^2}}$
Total head	$\frac{p}{\gamma} + \frac{V^2}{2g} + z =$ constant on a streamline $= H$