Midterm Exam

Math 53, October 5, 2016. Instructor: E. Frenkel

You have to solve the following 9 equally-weighted problems.

You must write your name and SID number at the top of every page.

Your solution to each problem must be written on the corresponding page. Please encircle your final answers.

You are not allowed to use any materials during the exam except for a list of formulas *handwritten* by yourself on *one side* of a standard size sheet of paper (*no printouts or photocopies allowed*). No calculators are allowed during the exam.

Justify your answers. You will not get full credit for an unsubstantiated or unjustified answer.

YOUR NAME: _____

YOUR SID: _____

NAME OF YOUR GSI: _____

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Problem 1. Consider the curve in $\mathbb{R}^2$ defined by the parametric equations

$$x = 1 + \ln t, \qquad y = -t^{-3}, \qquad t > 0.$$

Find a Cartesian equation for this curve, sketch the curve, and indicate with an arrow the direction in which the curve is traced as the parameter t is increasing.

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Problem 2. Sketch the curve $r = \sin 2\theta$ and find the area enclosed by it in the first quadrant.

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Problem 3. Consider the space curve defined by the parametric equations

$$x = t^3 - 1, \qquad y = t^2 + 2t, \qquad z = \ln t - \frac{1}{t}.$$

Find the cosine of the angle between the vector $\langle 1, 2, 2 \rangle$ and the tangent vector to this curve at the point of its intersection with the yz plane.

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Problem 4. Find parametric equations for the line of intersection of the planes $2x+5z+3 = 0$ and $x - 2y + z + 2 = 0$.

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Problem 5. Find an equation of the plane containing the points $(1, 2, 3)$, $(1, 1, 1)$, $(-2, 2, 4)$.

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Problem 6. Find an equation of the surface consisting of all points in $\mathbb{R}^3$ equidistant from the point $(0, -1, 0)$ and the plane $y = 1$. Sketch this surface (marking clearly the coordinate axes) and identify it by name.

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Problem 7. Write down an equation of the tangent plane to the surface

$$x^2 + 9y^2 - 3z^2 = 1$$

at the point $(2, 1, 2)$.

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Problem 8. Write down the differential and the linear approximation of the function

$$f(x, y, z) = \sqrt{x^2 + 4y^2 + z^2}$$

at $(3, 1, 6)$ and use it to approximate the number $\sqrt{(3.02)^2 + 4(0.99)^2 + (5.97)^2}$.

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Problem 9. Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^{10}y^{30}}{x^{20}+y^{60}}$$

does not exist.