Physics 7B

Midterm 2: Monday October 31st, 2016

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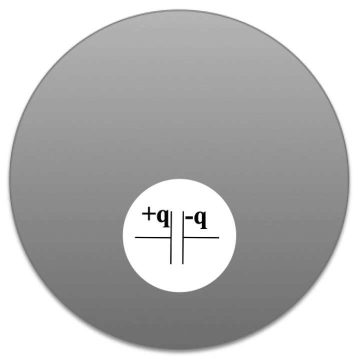
Total points: 100 (6 problems)

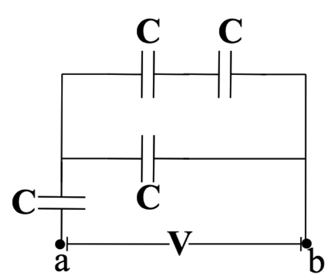
Show all your work and take particular care to explain what you are doing. Partial credit can be given. Please use the symbols described in the problems or define any new symbol that you introduce. Label any drawings that you make. ***Good luck!***

Problem 1: Conceptual Questions (15 pts)

(a) Suppose we have a solid conducting sphere with a cavity inside. Now, imagine we place a small, charged, parallel-plate capacitor with capacitance C inside the cavity, as shown below. What are the net charges on the internal and external surfaces of the sphere?

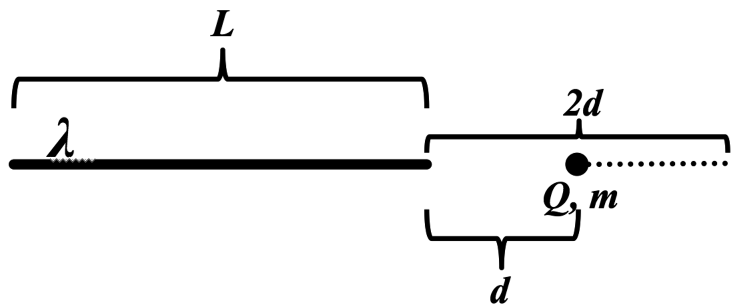


(b) In the figure below, all capacitors have the same capacitance C. Determine the equivalent capacitance between the points a and b.



Problem 2 (15 pts)

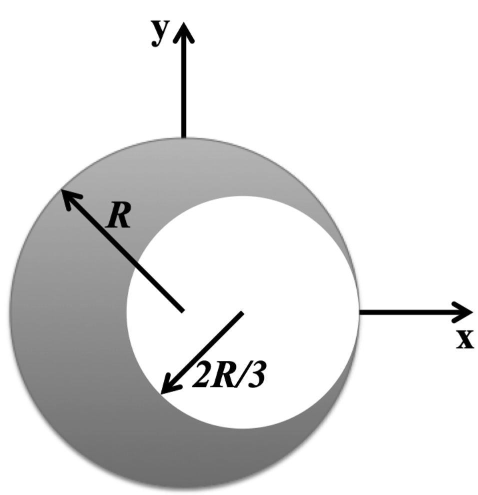
A positive charge is distributed uniformly with charge density λ along a line L-meters long. A positive charge Q with mass m is released from a point d-meters away from the end of the line, as shown below. Find the particle's velocity when it reaches a distance $2d$ from the end of the line charge.



Problem 3 (20 pts)

(a) Consider a sphere with uniform charge density ρ and radius R, centered at the origin. Let the electrostatic potential be $V = 0$ at infinity and find the potential at some radius r from the center of the sphere for $r < R$.

(b) Now consider the same sphere but with a cavity made by removing a sphere of radius $\frac{2R}{3}$ from the original sphere such that the cavity just reaches the outer surface at position $(R, 0, 0)$, as shown in the diagram below is a cross section of this 3D setup in the x-y plane. Use your answer from part (a) and the superposition principle to find the potential at the center of the cavity. Let the electrostatic potential be $V = 0$ at infinity.



Problem 4 (20 pts)

The plates of a parallel-plate capacitor have area A, separation x, and can be connected to a battery with voltage V. The capacitor is submerged in a liquid with dielectric constant K.

(a) The capacitor is charged by the battery, and then disconnected from it. The plates are then pulled apart until they are separated by $2x$. What are the initial and final energies stored in the capacitor?

(b) In this case, how much work is required to pull the plates apart (assume constant speed)?

(c) While connected to the battery, the plates are pulled apart until they are separated by $2x$. What are the initial and final energies stored in the capacitor?

(d) In this case, how much work is required to pull the plates apart (assume constant speed)?

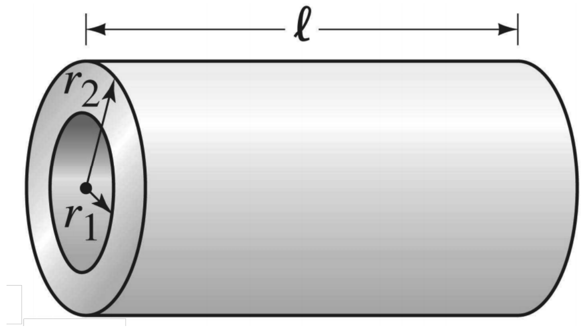
(e) In this case, calculate the change in energy of the battery.

Problem 5 (15 pts)

(a) Suppose a flat circular disk of radius R_0 has a nonuniform surface charge density of $\sigma = a/r$, where a is a constant and r is measured from the center of the disk. Let the x-axis be along the central axis of the disk with $x = 0$ at its center. Find the potential along the x-axis, $V(x)$ relative to $V = 0$ at $x = \infty$.

(b) What is the electric field along the x-axis, $\vec{E}(x)$?

Problem 6 (15 pts) A hollow cylindrical resistor with inner radius r_1, outer radius r_2, and length l, is made of a material with resistivity ρ, as shown below.



(a) Calculate the resistance for a current flowing parallel to the cylinder axis.

(b) Calculate the resistance R for a current that flows radially outward in terms of the given variables. (Hint: Divide the resistor into shells and integrate.)

Physics 7B, Fall 2016: Midterm 2 Formula Sheet

Electrostatics

$$\vec{F} = Q\vec{E}$$

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\epsilon = K\epsilon_0$$

$$C = KC_0$$

$$R = \rho \frac{l}{A}$$

$$P = IV$$

$$I = \int \vec{j} \cdot \vec{dA}$$

Mathematics

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n!\, 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\int (1+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (k^2+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{k^2+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (k^2+x^2)^{-1} dx = \frac{1}{k}\arctan\left(\frac{x}{k}\right)$$

$$\int (1+x^2)^{-\frac{3}{2}} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int (k^2+x^2)^{-\frac{3}{2}} dx = \frac{x}{k^2\sqrt{k^2+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2}$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left| tan\left(\frac{x}{2} + \frac{\pi}{4}\right) \right| \right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left| tan\left(\frac{x}{2}\right) \right| \right)$$