Problem 1

December 15, 2016

To melt all of the ice into water, we need $(10 \times 2 + 300)$ kJ, and the water can give off a heat of 400 kJ before turning into ice – thus, the final state is liquid water.
For this realization, assuming that you arrived there correctly, you get $\boxed{\text{3 points}}$.
Note that just getting a positive temperature at the end is not enough to determine this, because the formula we have to use assumes that the final phase is water.
So let's set up our equation. We set the heat lost by the water until it's cooled down to the final temperature T_f (on the left-hand-side of the equation) equal to the heat gained by the ice as it melts and warms up to T_f (on the right-hand-side of the equation):

$$10\ \text{kg} \cdot (4 \cdot 10^3) \frac{\text{J}}{\text{kg}^\circ\text{C}} \cdot (10^\circ\text{C} - T_f) = 1\ \text{kg} \left(2 \cdot 10^3 \frac{\text{J}}{\text{kg}^\circ\text{C}} \cdot (10^\circ\text{C}) + 3 \cdot 10^5 \frac{\text{J}}{\text{kg}} + 4 \cdot 10^3 \frac{\text{J}}{\text{kg}^\circ\text{C}} T_f \right)$$

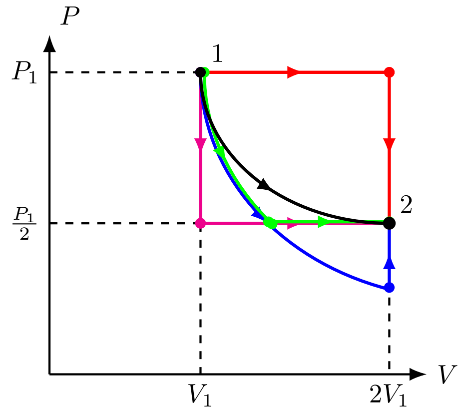
Solving for T_f gives

$$T_f = \frac{20}{11}\,^\circ\text{C} \approx 1.8^\circ\text{C}.$$

I would also accept 2°C as an answer, if it's derived and approximated correctly, without forgetting anything.
The general idea gives you $\boxed{\text{3 points}}$, getting the all the contributions right and the final result in the end gives you $\boxed{\text{4 points}}$. Many students lost points here because they didn't realize that the specific heat of the ice changes after it melted.
If you just had a small algebra mistake (that's not physics-related) that messed up your last result, you only get 1 point deducted, but still get the four points for the last part.

Solution

A free expansion is irreversible, adiabiatic ($Q = 0$) and isothermal ($T_1 = T_2 = T$). Note that only irreversible processes can be simultaneously adiabatic and isothermal. The initial state is (P_1, V_1, T) and the final state is $(2V_1, \frac{P_1}{2}, T)$ where P_2 follows from the ideal gas law. There are many possible reversible processes with the same initial and final state. I will show an exact solution to the most basic example: an isothermal expansion (**black** line). Note that by itself, a reversible adiabatic expansion cannot connect the initial and final state, as it would necessarily end up at a lower pressure or volume than an isothermal expansion with the same initial state. What is possible however, is a reversible adiabatic expansion to the final state volume followed by an isovolumetric temperature increase (blue line); or a reversible adiabatic expansion to the final state pressure followed by an isobaric expansion (green line).. Two other simple solutions are an isobaric expansion followed by an isovolumetric temperature decrease (red), or the same process in reverse order (magenta).



For a reversible isothermal expansion we have

$$dE = 0 = dQ - dW \tag{1}$$

$$\Delta S = \int \frac{dQ}{T} \tag{2}$$

$$= \int \frac{dW}{T} \tag{3}$$

$$= \int P \frac{dV}{T} \tag{4}$$

$$= \int_{V_1}^{2V_1} \frac{NkT}{V} \frac{dV}{T} \tag{5}$$

$$= Nk_B \ln\left(\frac{2V_1}{V_1}\right) \tag{6}$$

$$= Nk_B \ln(2) \tag{7}$$

Since entropy is a state variable, the entropy change ΔS_{12} only depends on the initial and final state 1 and 2 and not on the path between them. Therefore, the gas entropy change calculated for the isothermal expansion is the same for the irreversible free expansion, or for any other reversible or irreversible process between states 1 and 2. Entropy changes are **path independent**.

Rubric (max 10 points)

(a) -3 points for missing, faulty or too vague choice of reversible process

(b) -1 point for approximately correct process with minor error or omission

(c) -4 points for missing or completely faulty expression for ΔS

(d) for partially correct ΔS:

- **(i)** -1 point for algebraic or conceptual error in derivation
- **(ii)** -1 point for missing factor of Nk_B (or nR)

- **(iii)** -1 point for result with temperature dependence
- **(iv)** -1 point for result not proportional to ln(2)

(e) -3 points for missing or faulty statement on path indepence of entropy

(f) -1 point for incomplete statement on path independence

Physics 7B Prof. Birgeneau Final #3

Matthew Quenneville

December 15, 2016

The straight wire segments will not contribute since $d\vec{\ell}$ and $\hat{r}$ are parallel/anti-parallel along these paths. Thus, by Biot-Savart:

$$\begin{aligned}
\vec{B} &= \frac{\mu_0 I}{4\pi}\int_{\text{circle}} \frac{d\vec{\ell}\times\hat{r}}{r^2} \\
&= \frac{\mu_0 I}{4\pi}\int_0^{\pi/2} \frac{Rd\phi}{R^2}(\hat{\phi}\times\hat{r}) \\
&= \frac{-\mu_0 I}{4\pi R}\int_0^{\pi/2} d\phi\hat{z} \\
&= \frac{-\mu_0 I}{8R}\hat{z}.
\end{aligned} \tag{1}$$

Physics 7B Prof. Birgeneau Final #4 Solution

Matthew Quenneville

December 15, 2016

For gaussian cylinder centered on the symmetry axis, with radius r and length L, we have:

$$\oint \vec{E} \cdot d\vec{a} = E_r \int da = E_r \cdot 2\pi r L. \tag{1}$$

Note that the electric field must point radially away from (or towards) the z-axis by symmetry. Now apply Gauss's law, $\oint \vec{E} \cdot d\vec{a} = Q_{\text{enc}}/\epsilon_0$ in each case.

(a) For $r < R_1$, $Q_{\text{enc}} = 0$. Thus,

$$E_r = 0, \tag{2}$$

and thus,

$$\vec{E} = 0 \tag{3}$$

(b) For $R_1 < r < R_2$, $Q_{\text{enc}} = QL/\ell$. Thus,

$$E_r = \frac{Q}{2\pi r \ell \epsilon_0}, \tag{4}$$

and thus,

$$\vec{E} = \frac{Q\hat{r}}{2\pi r \ell \epsilon_0}. \tag{5}$$

(c) For $R_2 < r$, $Q_{\text{enc}} = 0$. Thus,

$$E_r = 0, \tag{6}$$

and thus,

$$\vec{E} = 0. \tag{7}$$

(d)

$$\vec{E}(r = R_{\text{el}}) = \frac{Q\hat{r}}{2\pi R_{\text{el}}\ell\epsilon_0} \tag{8}$$

For circular motion:

$$F = \frac{mv^2}{r}. \tag{9}$$

Since only electrostatic forces act on the particle,

$$F = |q\vec{E}|. \tag{10}$$

Thus,

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 \\ &= \frac{R_{\text{el}}F}{2} \\ &= \frac{R_{\text{el}}|qE|}{2} \\ &= \frac{|qQ|}{4\pi\ell\epsilon_0}. \end{aligned} \tag{11}$$

(e) Use Ampere's Law, $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$. Thus,

$$B(\vec{r}) = \frac{\mu_0 I\hat{\phi}}{2\pi r}. \tag{12}$$

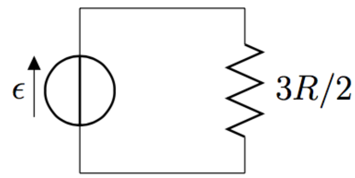
Circular motion $\implies \vec{v} = v\hat{\phi}$. Thus, the force is given by:

$$\begin{aligned} \vec{F} &= q\vec{v} \times B(\vec{r}) \\ &= qvB\hat{\phi} \times \hat{\phi} \\ &= 0. \end{aligned} \tag{13}$$

Problem 5

(a) 3pts

The potential across a uncharged capacitor is zero, so we can ignore it. The following is the equivalent circuit.



Grading Rubric:
+1.5 Correct equivalent resistance
+1.5 Ignoring capacitor

(b) 3pts

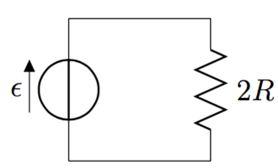
The current through the voltage source is $I_{battery} = I_1 = \frac{\mathcal{E}}{R_{eq}} = \frac{2\mathcal{E}}{3R}$
By symmetry, $I_2 = I_3$. Using Kirchhoff's current rule, $I_1 = I_2 + I_3$. Thus, $I_2 = I_3 = \frac{1}{2}I_1 = \frac{\mathcal{E}}{3R}$.

Grading Rubric:
+1 Correct I_1
+1 Correct I_2
+1 Correct I_3
+1 Wrong I_2 and I_3, but showed that $I_2 = I_3$

(c) 3pts

As $t \to \infty$, the capacitor is fully charged, and no current flows through that part of the circuit ($I_3 = \frac{dQ}{dt} \to 0$), so we ignore it. The equivalent circuit is the following:

Grading Rubric:
+1.5 Correct equivalent resistance
+1.5 Ignoring capacitor



(d) 3pts

As explained, $I_3 = 0$. By symmetry, $I_1 = I_2 = \frac{\mathcal{E}}{2R}$

Grading Rubric:

Problem 1 [(d) 3pts] continued on next page. . .

+1 Correct I_1
+1 Correct I_2
+1 Correct I_3
+1 Wrong I_1 and I_2, but showed that $I_1 = I_2$

(e) 3pts

Applying the Kirchhoff's loop rule, $-I_3R - V_{cap} + I_2R = 0$. Hence, the voltage across the capacitor $V_{cap} = I_2R = \frac{\mathcal{E}}{2}$.

Grading Rubric:
+1 Correctly applying Kirchhoff's Loop Rule ($-I_3R - V_{cap} + I_2R = 0$)
+1.5 Showing $V_{cap} = I_2R$
+0.5 Correct Answer
(+1.5 Knowing $V_{cap}(t \to \infty) = \frac{Q(t\to\infty)}{C}$

Problem 6

A circular loop of wire of radius a is placed in a uniform magnetic field, with the plane of the loop perpendicular to the field. The field varies over time with a functional form $B(t) = B_0 + bt^2$, where B_0 and b are positive constants.

(a) Calculate the magnetic flux through the loop at $t = 0$.

(b) Calculate the induced emf in the loop as a function of time.

(c) What is the induced current as a function of time and its flow direction if the overall resistence of the loop is R, assuming the $\boldsymbol{B}$-field points out of the page?

(d) Find the power dissipated due to the resistance of the loop as a function of time.

(e) Now suppose the loop is made from a material in a superconducting state. What is the power dissipated by the resistance of the loop as a function of time?

Solution.

(a) The magnetic flux through the loop is

$$\Phi_B(t=0) = \int_{\text{inside loop}} \boldsymbol{B}(t=0) \cdot d\boldsymbol{A} = B(t=0)\pi r^2 = B_0 \pi a^2.$$

(b) The induced EMF is given by Faraday's Law:

$$\mathcal{E} = -\frac{d}{dt}\Phi_B(t) = -\frac{d}{dt}B(t)\pi a^2 = -2\pi a^2 B_0 bt.$$

(Note: since we haven't yet specified which way the field points, either sign is acceptable here.)

(c) Lenz's law says that "nature hates a change in flux". Since the magnetic field is pointing out of the page and increasing in magnitude, an induced current will be created whose magnetic field *opposes* the change in flux. Such a current must run *clockwise*. We can then find the induced current via Ohm's law:

$$I(t) = \frac{\mathcal{E}}{R} = \frac{-2\pi a^2 B_0 t}{R}.$$

Again, this is *clockwise*.

(d) Power dissipated is given by

$$P(t) = I\mathcal{E} = \frac{4\pi^2 a^4 B_0^2 t^2}{R}.$$

(e) In a superconductor, resistance is exactly zero, so no energy is dissipated at any time.

1 Starting Rubric

(a) **(5)**

- **(1)** Correctly state Faraday's Law:

$$\oint_{\text{loop}} \boldsymbol{E} \cdot \mathrm{d}\,\boldsymbol{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \tag{1}$$

- **(1)** Correctly find $\Phi_B(t)$
- **(1)** Correctly find $\mathcal{E} = -\mathrm{d}\Phi_B/\mathrm{d}t$
- **(1/2)** Use symmetry argument to justify pulling $\boldsymbol{E}$ from the integral
- **(1.5)** Correct final result. Lose 0.5 points for incorrect or neglected direction of $\boldsymbol{E}$, and lose 1 point for using $\boldsymbol{E} = -\nabla V$

(b) **(5)**

- **(2)** Get that $\Phi_B(t) = \pi r_0^2 B(t)$ around a loop centered on the origin with $r > r_0$
- **(1)** Correctly find $\Phi_B(t)$ from $\Phi_B(t)$
- **(1/2)** Use symmetry argument to justify pulling $\boldsymbol{E}$ from the integral
- **(1.5)** Correct final result, or result consistent with minor algebra mistakes. Lose 0.5 points for incorrect or neglected direction of $\boldsymbol{E}$ and lose 1 point for using $\boldsymbol{E} = -\nabla V$

(c) **(5)**

- **(1/2)** Reason why $\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$ is not zero
- **(1)** State

$$W = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{x}$$

- **(1/2)** Relate $\boldsymbol{F} = q\boldsymbol{E}$
- **(2)** Obtain explicit solution

$$W = q \oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$$

  for $\boldsymbol{E}$ found in part (a) and (b)
- **(1)** Show explicitly how this integral is nonzero in terms of variables in the problem.

Paritial credit may be given for each of these sections.

2 Solution

(a) Faraday's Law tells us

$$\oint_{\text{loop}} \boldsymbol{E} \cdot \mathrm{d}\,\boldsymbol{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \tag{2}$$

Using the left-hand side of equation 2 to get the $\boldsymbol{E}$-field requires us to know some symmetry it must have. Here, because the problem is azimuthally symmetric, and we see there is a potential $\mathcal{E} = -\mathrm{d}\Phi_B/\mathrm{d}t$ that builds up around a circular loop around the center of the magnet, we can see that $\hat{\boldsymbol{E}} = \hat{\boldsymbol{\theta}}$. Also, we see that $\boldsymbol{E}$ is independent of θ and, in between the poles of the magnet, it is also independent of z. Thus,

$$\boldsymbol{E}(\boldsymbol{r}) = E(r)\boldsymbol{\theta}.$$

This motivates our choice of a circular loop of radius r, with path $\mathrm{d}\boldsymbol{s} = r\mathrm{d}\theta\hat{\boldsymbol{\theta}}$. Then, the left-hand side of 2 is

$$\oint_{\text{loop}} \boldsymbol{E} \cdot \mathrm{d}\,\boldsymbol{s} = \int_0^{2\pi} (E(r)\boldsymbol{\theta}) \cdot \left(r\mathrm{d}\theta\hat{\boldsymbol{\theta}}\right) \tag{3}$$

$$= 2\pi E(r) \tag{4}$$

Now, calculating the right-hand side (RHS) of Faraday's law, the magnetic flux Φ_B through our loop is

$$\begin{aligned}\Phi_B(t) &= \int \boldsymbol{B}(t)\cdot \mathrm{d}\,\boldsymbol{A} &(5)\\ &= \int_0^{2\pi}\int_0^{r} (B(t)\hat{\boldsymbol{z}})\cdot\left(r\mathrm{d}r\mathrm{d}\theta\hat{\boldsymbol{\theta}}\right) &(6)\\ &= B(t)\pi r^2 &(7)\end{aligned}$$

Therefore,

$$-\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}B}{\mathrm{d}t}\pi r^2. \tag{8}$$

Equating equations 4 and 7 and solving for $E(r)$, $\boldsymbol{E}$ is

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{r}{2}\frac{\mathrm{d}B}{\mathrm{d}t}\hat{\boldsymbol{\theta}} \tag{9}$$

(b) The LHS is the same for this problem, but the magnetic flux becomes

$$\begin{aligned}\Phi_B(t) &= \int_0^{2\pi}\int_0^{r} (B(t)\hat{\boldsymbol{z}})\cdot\left(r\mathrm{d}r\mathrm{d}\theta\hat{\boldsymbol{\theta}}\right) &(10)\\ &= B(t)\pi r_0^2 &(11)\end{aligned}$$

because the contributions for $r > r_0$ are 0, as $\boldsymbol{B}\,(r > r_0) = 0$. Then $\mathcal{E} = -\pi r_0^2 \mathrm{d}B/\mathrm{d}t$ and Faraday's law gives $\boldsymbol{E}$ for $r > r_0$ as

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{r_0^2}{2r}\frac{\mathrm{d}B}{\mathrm{d}t}\hat{\boldsymbol{\theta}} \tag{12}$$

(c) Showing the force $\boldsymbol{F} = q\boldsymbol{E}$ is not conservative is equivalent to showing the work around a closed loop is nonzero. The work done by any force is

$$W = \int \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{x}.$$

The work done by the electric field on a hypothetical charge q around a closed loop of radius r around the origin is

$$W = \int_0^{2\pi} (q\boldsymbol{E})\cdot\left(r\mathrm{d}\theta\hat{\boldsymbol{\theta}}\right) \tag{13}$$

Plugging in for $\boldsymbol{E}$ for the two cases, we see

$$\begin{aligned}W(r < r_0) &= \frac{q}{2}\frac{\mathrm{d}B}{\mathrm{d}t}r^2\int_0^{2\pi}\mathrm{d}\theta = \pi r^2 q\frac{\mathrm{d}B}{\mathrm{d}t} &(14)\\ W(r > r_0) &= \frac{q}{2}\frac{\mathrm{d}B}{\mathrm{d}t}r_0^2\int_0^{2\pi}\mathrm{d}\theta = \pi r_0^2 q\frac{\mathrm{d}B}{\mathrm{d}t} &(15)\\ &\neq 0 &(16)\end{aligned}$$

Thus we see the force from the electric fields is not conservative.

Physics 7B - Lecture 1 Final Exam Q8 Rubric

James Reed Watson

December 15, 2016

a) (4 points) When the swith is flipped initially, the inductor blocks current from flowing. Inductors resist changes in current by producing a back emf. Thus, the effective resistance of the cicuit is simply the resistance of R_1 and R_2 in series. So the current is

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{\mathcal{E}}{2R} \tag{1}$$

$$I_3 = 0 \tag{2}$$

b) (4 points) After a long period of time, the inductor has no back emf, and acts like a wire. As a result, we must solve the equation for series *and* parallel resisters. The results are:

$$I_1 = \frac{\mathcal{E}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{2\mathcal{E}}{3R} \tag{3}$$

$$I_2 = \frac{\mathcal{E}}{R_2}[1 - \frac{R_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}] = \frac{\bullet E}{3R} \tag{4}$$

$$I_3 = \frac{\mathcal{E}}{R_3}[1 - \frac{R_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}] = I_2 \tag{5}$$

c) (7 points) The decay constant for an inductor is L/R. When the switch is disconnected, the current through the inductor will initially be the same as before, but now it is in series with R_2 and R_3. Thus the decay constant is $\tau = \frac{L}{R_2 + R_3} = \frac{L}{2R}$. As a result, we use the previous result, and add in an exponential decay to obtain

$$-I_2 = I_3 = \frac{\mathcal{E}}{3R} \exp[-2tR/L] \tag{6}$$

$$I_1 = 0 \tag{7}$$

Note the minus sign on I_2 which results from the current running in the opposite direction as the convention specified.

- a) 1 points for recognizing that the inductor acts like a break, or plugging in t=0 into the explicit formula.
- a) 1.5 points for writing down any equation for kirchoff's laws or equivalent resistance.
- a) .5(x3) points for the correct values on each current.
- b) 1 point for recognizing that the inductor will act like a wire, or pluggin in $t = \infty$ into the explicit formula.
- b) .5(x3) points for correct values on each current.
- c) 2 points for having an exponential in the answer
- c) 1 point for the right time constant $\tau = \frac{L}{2R}$
- c) 1 point for having the right maximum current $I = \frac{\mathcal{E}}{3R}$
- c) 1(x3) points for each correct current.