Physics H7B – Spring 2002 (Bale)
Midterm Exam #1
One 3x5 note card is allowed
March 5, 2002

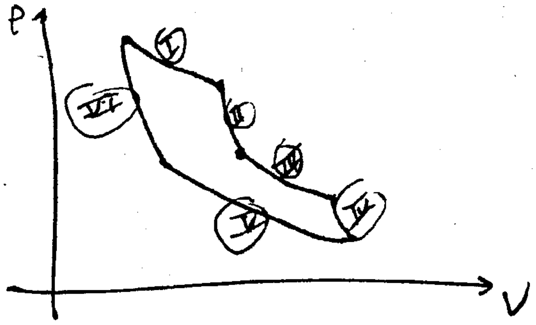
Problem 1 (25 points)

A system absorbs 200 J of energy from a reservoir at 400 K and also 300 J from a reservoir at 300 K. It interacts with a third reservoir whose temperature is T_3. When the system returns to its original state, it has done 100 J of work.

1) What is the entropy change of the system for the complete cycle?
2) Assuming that the cycle is reversible, draw and label the P-V diagram.
3) What is T_3?

1) $\Delta S = 0$ since system returns to original state

2)



(I) isothermal expansion at 400K

$\rightarrow \Delta S = \frac{200J}{400K} = \frac{Q_I}{T_I}$

(II) adiabatic expansion

(III) isothermal expansion at 300K

$\Delta S = \frac{300J}{300K} = \frac{Q_{III}}{T_{III}}$

(IV) adiabatic expansion

(V) isothermal compression

$\Delta S = \frac{Q_3}{T_3}$

(VI) adiabatic compression

3) so $\Delta S = \frac{Q_I}{T_I} + \frac{Q_{III}}{T_{III}} + \frac{Q_3}{T_3} = 0$

since $\Delta T = 0 \rightarrow \Delta U = 0 = Q + W = Q_I + Q_{III} + Q_3 - W = 0$

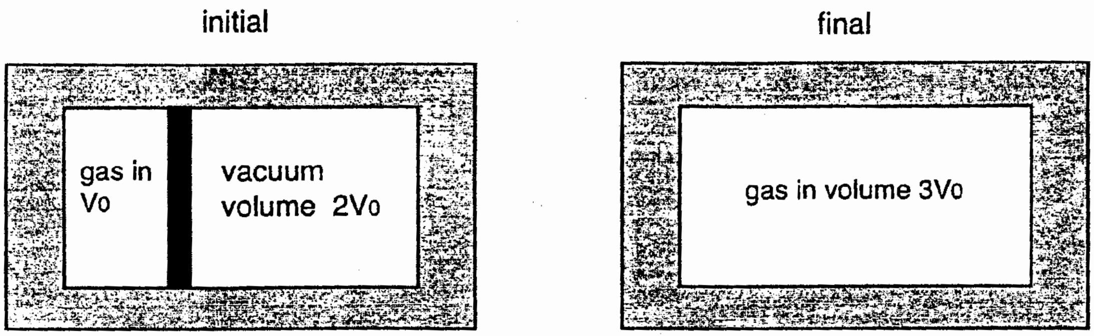
so $Q_3 = W - Q_I - Q_{III} = 100J - 200J - 300J$

$Q_3 = -400J$

So $\frac{200J}{400K} + \frac{300J}{300K} - \frac{400J}{T_3} = 0$

$\rightarrow T_3 = 267K$

Problem 2 (20 points)



A monatomic gas is contained in a volume V_0, in an insulated container. A partition is removed and the gas expands to fill an additional volume $2V_0$. Calculate the change of entropy.

No heat flows through walls, so $dT = dU = 0$

$T = \text{const}$

$\Delta S = \frac{dQ}{T}$ but $dQ = dW = p\,dV$

$\Delta S = \frac{dQ}{T} = \frac{p\,dV}{T}$ ideal gas $pV = nk_BT$

$\downarrow$ $n =$ number density

$\Delta S = \frac{nk_BT}{V}\frac{dV}{T}$ $p = \frac{nk_BT}{V}$

so $S_{final} - S_{initial} = nk_B \int_{V_0}^{3V_0} \frac{dV}{V}$

$= nk_B \ln V \Big]_{V_0}^{3V_0} = nk_B \ln \frac{3V_0}{V_0} = nk_B \ln 3$

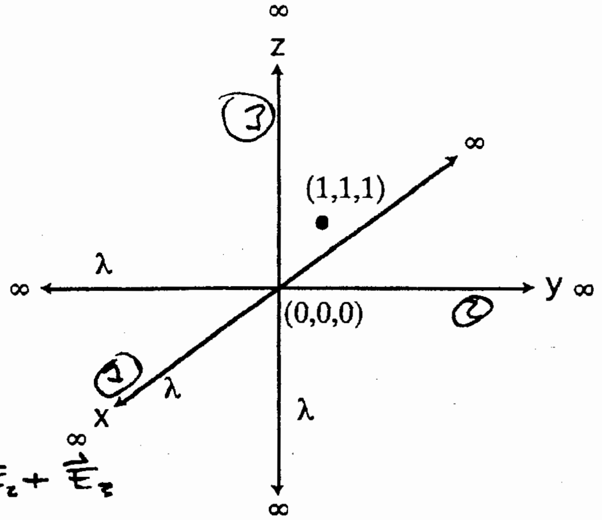
so $\boxed{S_f - S_i = nk_B \ln 3}$ or $NR \ln 3$ $N =$ # of moles

$R =$ gas constant

note entropy increases — free expansion of gas is irreversible!

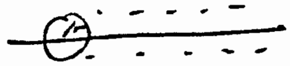
Problem 3 (25 points)

Three infinite line charges, each with linear charge density λ, meet at the origin (0,0,0) and are orthogonal to one another. Calculate the electric field magnitude and direction at the point (1,1,1).



Use Gauss' Law and superposition for each line charge

(10 pts)



$$\oint \vec{E}\cdot d\vec{A} = 4\pi Q_{enc}$$

$$E\cdot 2\pi r L = 4\pi \lambda L$$

$$\rightarrow \vec{E} = \frac{2\lambda}{r}\hat{r}$$

(5 pts)

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

(1) charge along $\hat{x}$: $\hat{r} = \frac{y}{r}\hat{y} + \frac{z}{r}\hat{z}$ so $\vec{E}_1 = \frac{2\lambda}{y^2+z^2}(y\hat{y} + z\hat{z})$

$$\vec{E}_1(1,1,1) = \lambda(\hat{y}+\hat{z})$$

(2) charge along $\hat{y}$: $\hat{r} = \frac{x}{r}\hat{x} + \frac{z}{r}\hat{z}$ so $\vec{E}_2 = \frac{2\lambda}{(x^2+z^2)}(x\hat{x} + z\hat{z})$

$$\vec{E}_2(1,1,1) = \lambda(\hat{x}+\hat{z})$$

(3) charge along $\hat{z}$: $\hat{r} = \frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}$ so $\vec{E}_3 = \frac{2\lambda}{(x^2+y^2)}(x\hat{x} + y\hat{y})$

$$\vec{E}_3(1,1,1) = \lambda(\hat{x}+\hat{y})$$

so $\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$ at (1,1,1)

$$= \lambda(\hat{y}+\hat{z}) + \lambda(\hat{x}+\hat{z}) + \lambda(\hat{x}+\hat{y})$$

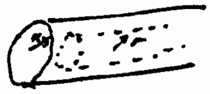
$$\boxed{\begin{aligned} \vec{E} &= 2\lambda(\hat{x}+\hat{y}+\hat{z}) \\ |E| &= \sqrt{(2^2+2^2+2^2)\lambda^2} = \sqrt{12}\,\lambda = 2\sqrt{3}\,\lambda \\ \hat{E} &= \frac{1}{\sqrt{3}}(\hat{x}+\hat{y}+\hat{z}) \end{aligned}}$$

Problem 4 (30 points)

A coaxial cable carries a volume charge ρ on its inner cylinder (radius α) and a surface charge σ on the outer cylinder (radius β). The charges balance so that the total charge is zero. Calculate the electric field in the regions 1) $r < \alpha$, 2) $\alpha < r < \beta$, and 3) $r > \beta$. Plot E against r.

Use Gauss' Law with cylindrical symmetry (10 pts)

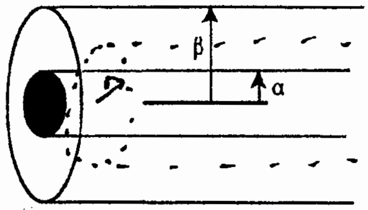
for $r<\alpha$: Gaussian surface within α



$$\oint E\cdot dA = 4\pi Q_{enc}$$

$$E\cdot 2\pi r L = 4\pi(\rho \pi r^2 L)$$

$$\rightarrow \boxed{E = 2\pi\rho r} \quad \text{(5 pts)}$$



for $\alpha<r<\beta$: Gaussian surface between α & β

$$\oint E\cdot dA = 4\pi Q_{enc}$$ ← now entire inner cylinder is enclosed

$$E\cdot 2\pi r L = 4\pi(\rho \pi \alpha^2 L)$$

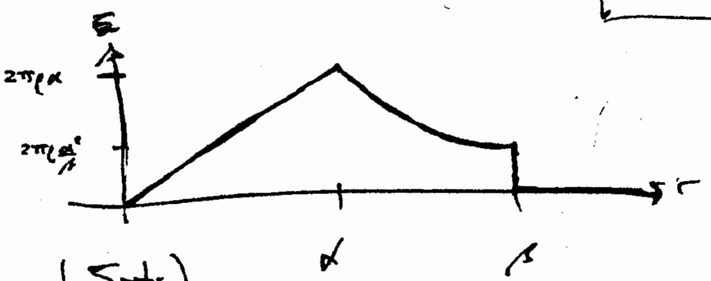
$$\rightarrow \boxed{E = 2\pi\rho \frac{\alpha^2}{r}} \quad \text{(5pts)}$$

for $r>\beta$: Gaussian surface outside coax

since total charge = 0

$$\oint E\cdot dA = 0 \rightarrow \boxed{E = 0} \quad \text{(5 pts)}$$

So



(5pts)