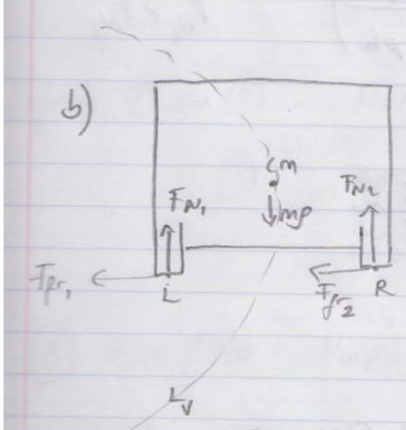
· Car flipping over

a) $v = v_{skid}$ $\qquad F_{fr_{max}} = F_c$ $\qquad N = mg$

$$\mu_s \cdot \cancel{m}g = \frac{\cancel{m}v^2}{R}$$

$$v_{skid} = \sqrt{\mu_s g R} \qquad v_{skid}^2 = \mu_s g R$$

b)




$v = v_{tip}$ when $F_{N_1} = 0$

① $F_{N_2} = mg$

② $F_{fr_2} = \frac{m v_{tip}^2}{R}$

③ $\sum \tau_{cm} = 0$ $\qquad F_{fr_2} \cdot \alpha h = F_{N_2} \cdot b/2$

$$\frac{\cancel{m} v_{tip}^2}{R} \alpha h = \cancel{m}g \cdot b/2$$

$$v_{tip}^2 = \frac{Rgb}{2\alpha h}$$

c) $v_{skid} < v_{tip}$

$$\mu_s g \cancel{R} < \frac{\cancel{R}\cancel{g}b}{2\alpha h}$$

$$\boxed{\alpha < \frac{b}{2h\mu_s}}$$

Problem 2

a) The merry-go-around can be approximated as a uniform disk with mass $m_2$ and radius $R$.

We can compile the information as follows

| Object | Mass | Distance from axis | Moment of Inertia |
|---|---|---|---|
| Merry-go-around | $m_2$ | $0 < r < R$ | $\frac{1}{2} m_2 R^2$ |
| Child | $m_1$ | $R$ | $m_1 R^2$ |
| Ball | $m_3$ | $R$ | $m_3 R^2$ |
| | | | Total = sum of the above. |

Before the child catches the ball, $I_i = \frac{1}{2} m_2 R^2 + m_1 R^2$.

After the child catches the ball, $I_f = \frac{1}{2} m_2 R^2 + m_1 R^2 + m_3 R^2$.

b) Angular momentum about the center of the merry-go-around is conserved. $L_i = L_f$.

The initial angular momentum is given by $Rv\cos\phi\, m_3$.

The final angular momentum is given by $I_f \omega$.

Therefore, the final angular velocity is $\omega = \frac{Rv\cos\phi\, m_3}{I_f}$.

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Problem 3

Velocity of water in the main drainage pipe,

$$v_M = \frac{kwl}{\pi R^2}$$

Pressure of water in the main drainage pipe,

$$p_M = p_{atm} + \rho g h_2$$

where $\rho$ is the density of water.

Use Bernoulli's equation for the top of the downspout and the main drainage pipe,

$$p_{atm} + \rho g h_1 = p_M + \frac{1}{2}\rho v_M^2$$

Substitute $v_M$ and $p_M$ into the above equation, we get

$$k = \frac{\pi R^2}{wl}\sqrt{2g(h_1 - h_2)}$$

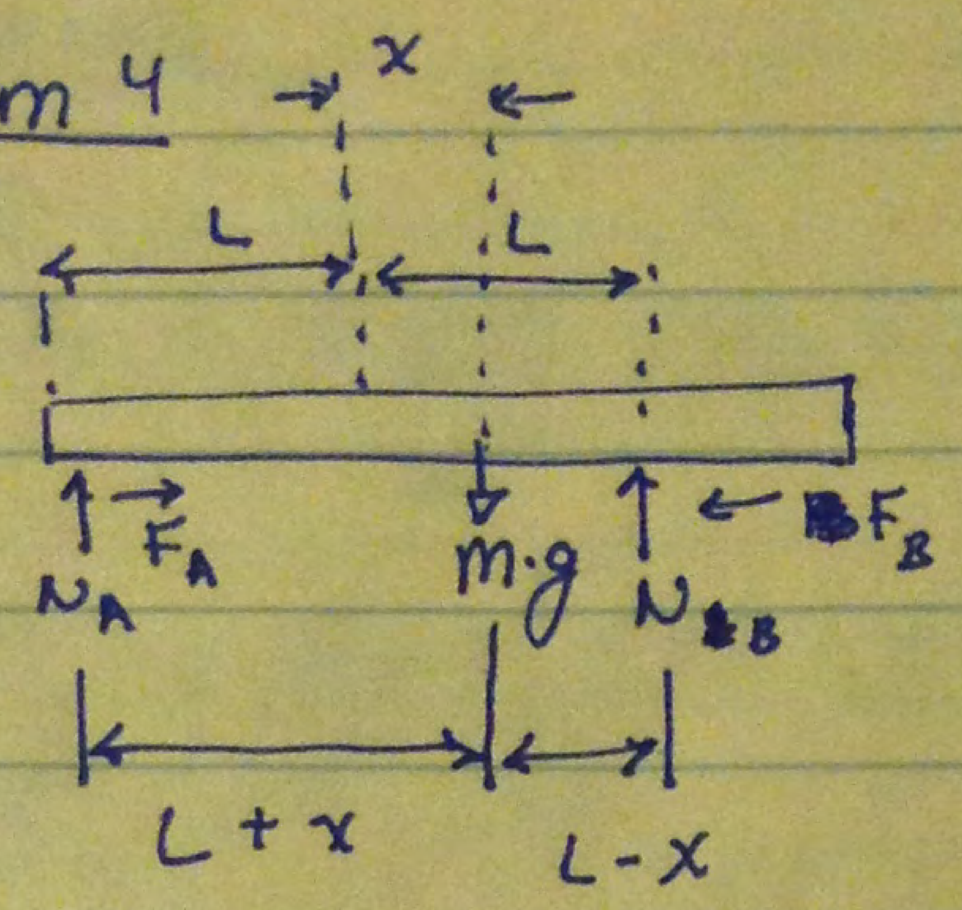
Problem 6

a) If we take a big sip, the length of the air column in the bottle increases, the wavelength of standing wave increases, the frequency decreases.

b) The fundamental frequency of an open pipe is $f_{open} = \frac{v}{2l}$, where $v$ is the velocity of sound in air, $l$ is the length of the pipe. The fundamental frequency of a closed pipe is $f_{closed} = \frac{v}{4l}$.

$$\frac{f_{closed}}{f_{open}} = \frac{1}{2}$$

problem 4




part a

$$\circlearrowleft + \; \sum M_A = 0 = g \cdot m \cdot (L + x) - N_B \cdot 2L$$

$$gm\,(L + x) = N_B\, 2L$$

$$N_B = \frac{gm}{2L}(L + x) = \frac{gm}{2}\left(1 + x/L\right)$$

$$\boxed{N_B = \frac{gm}{2}\left(1 + x/L\right)}$$

$$+\uparrow \sum F_y = 0 = N_A + N_B - mg$$

$$N_A = m \cdot g - N_B = mg - \frac{gm}{2}\left(1 + x/L\right)$$

$$N_A = \frac{mgL}{L} - \frac{gm}{2}\frac{L}{L} - \frac{gm}{2L}x = \frac{mgL}{2L} - \frac{mgx}{2L}$$

$$\boxed{N_A = \frac{mg}{2}\left(1 - x/L\right)}$$

part b

$$F_A = \mu N_A = \frac{\mu mg}{2}\left(1 - x/L\right)$$

$$F_B = \mu N_B = \frac{\mu mg}{2}\left(1 + x/L\right)$$

problem # 4

part c

$\sum F_x = m\frac{d^2x}{dt^2} = -F_B + F_A$

$$\cancel{m}\frac{d^2x}{dt^2} = \frac{\cancel{m}gr}{2}\left[\left(\cancel{1} - \frac{x}{L}\right) - \left(\cancel{1} + \frac{x}{L}\right)\right]$$

$$\frac{d^2x}{dt^2} = -\frac{gr}{\cancel{2}} \cdot \frac{\cancel{2}x}{L}$$

or,

$$\boxed{\frac{d^2x}{dt^2} + \frac{rg}{L}x = 0}$$

thus, we see the solution will be in the form of a simple harmonic oscillation since the ode takes the form $\frac{d^2x}{dt^2} + Ax = 0$.

part d

$$\omega^2 = \frac{rg}{L} \longrightarrow \omega = \sqrt{\frac{rg}{L}}$$

$$\& \quad T = \frac{2\pi}{\omega} \longrightarrow \boxed{T = 2\pi\sqrt{\frac{L}{rg}}}$$

part e

Initial conditions

1. at $t=0$ $x = x_0$
2. at $t=0$ $dx/dt = 0$

problem 4

① $x = A\cos(\omega t + \phi)$

② $\frac{dx}{dt} = A\omega \sin(\omega t + \phi)$

applying our initial conditions this reduces to.

1a $x_0 = A\cos(\cancel{\omega} 0 + \phi)$

2a $0 = A\omega \sin(0 + \phi)$

to satisfy 2a $\phi = 0$, thus 1a reduces to

$x_0 = A \cancel{\cos(0)}' \Rightarrow x_0 = A$

$\therefore$ $\boxed{\begin{matrix} \phi = 0 \\ A = x_0 \end{matrix}}$

part f

$$x(t) = x_0 \cos\left(\sqrt{\frac{rg}{L}}\, t\right)$$

part g it will fly out.

PROBLEM 5

a) TO THE RIGHT. A TRANSVERSE WAVE CARRIES MOMENTUM IN ITS DIRECTION OF TRAVEL. CONSERVATION OF MOMENTUM TELLS US THE WAVE MUST EXCITE A NEW WAVE WITH MOMENTUM GOING TO THE RIGHT.

b) $f_2 = f_1$ SINCE FREQUENCY IS INDEPENDENT OF MEDIA.

c) $v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{T}{3\mu_1}} = \frac{1}{\sqrt{3}} v_1$

d) $\lambda_2 = \frac{v_2}{f_2} = \frac{1}{\sqrt{3}} \frac{v_1}{f_1} = \frac{1}{\sqrt{3}} \lambda_1$

e) $A_T = \frac{2\lambda_2}{\lambda_1 + \lambda_2} A$ (SEE APPENDIX)

$$= \frac{2}{1+\sqrt{3}} A$$

f) $A_R = A - A_T$

$$= \left(1 - \frac{2}{1+\sqrt{3}}\right) A$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} A$$

IN THE SAME MEDIA, $E \propto A^2$, SO

$$\frac{E_R}{E} = \frac{A_R^2}{A^2}$$

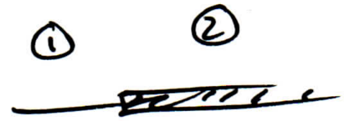
$$= \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$$

$$\approx 0.0718$$

PROBLEM 5

APPENDIX

TO GET TRANSMISSION COEFFICIENT, USE BOUNDARY CONDITIONS.




$$y_1 = A\sin(k_1x - \omega_1t) + A_R\sin(k_1x + \omega_1t)$$

$$y_2 = A_T\sin(k_2x - \omega_2t)$$

$$y_1(0,t) = y_2(0,t)$$

$$\Rightarrow A - A_R = A_T$$

$$\frac{dy_1}{dx}\bigg|_{x=0} = \frac{dy_2}{dx}\bigg|_{x=0}$$

$$\Rightarrow k_1A + k_1A_R = k_2A_T$$

SOLVING FOR $A_T$

$$A_T = \frac{2k_1}{k_1+k_2}A$$

$$= \frac{2\lambda_2}{\lambda_1+\lambda_2}A$$

USING $k = 2\pi/\lambda$.