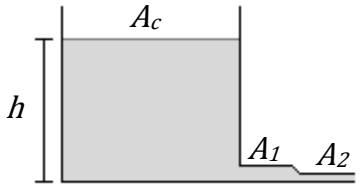
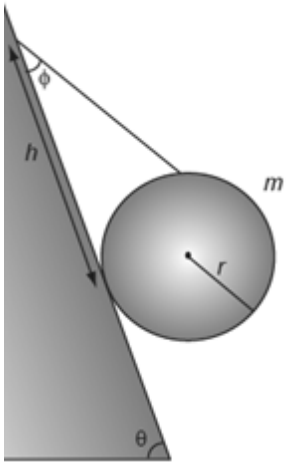
PHYS 7A, Lecture 002, Yildiz
Final Exam, Fall 2010
14 December 2010

1. (20 pts) If a billiard ball is hit in just the right way by a cue stick, the ball will leave the cue stick with a center of mass speed v_0 and a reverse spin of angular speed ω_0. Consider a billiard ball (radius r, mass M) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force F on the ball for a time t at a point that is a height h above the table's surface (see the figure). Assume that the coefficient of kinetic friction between the ball and table is μ_k.
   a. If ω_0 is equal to $2v_0/r$, find h in terms of F, μ_k, r, g, and M.
   b. Kinetic friction acts on the ball as it skids across the table. Determine the final linear speed of the ball (both direction and magnitude) as it rolls without slipping.

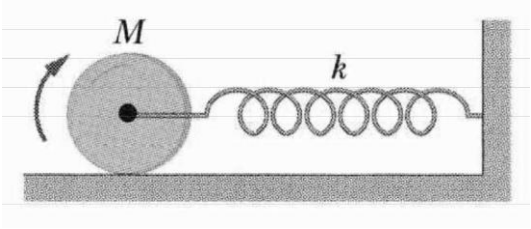
2. (15 pts) Consider a cylindrical tank of water, which has cross-sectional area $A_c = 100\ cm^2$. A hose is connected to a hole at the bottom of the tank, which allows water to flow out and which is small compared to the size of the tank. Where the hose is connected to the tank, it has cross-sectional area $A_1 = 4\ cm^2$ and at the end has $A_2 = 1\ cm^2$. Water fills the tank to a height h. The density of water is ρ, and gravitational acceleration is g.
   a. What are the velocities of the water v_1 in the section of the hose with $A_1 = 4\ cm^2$ and v_2 in the section with $A_2 = 1\ cm^2$? Neglect for parts a. and b. the movement of the water at the top of the tank as it drains.
   b. What are the pressures P_1 in the section of the hose with $A_1 = 4\ cm^2$ and P_2 at the point where the water exits the hose?
   c. If the height of the water is $h = 10\ cm$, $\rho = 1000\ kg/m^3$, and $g = 9.8\ m/s^2$, how long does it take for the tank to drain (numerical answer)? You may use the answers from the previous parts in your calculation.



3. (15 pts) A uniform sphere of mass m and radius r is tethered to a wall by a rope. The rope is tied to the wall a distance h above the contact point of the sphere, as shown. The rope makes angle ϕ to the wall and is *not* in line with the ball's center. The wall makes angle θ to the ground, all as shown in the figure below. Suppose the sphere is just on the verge of slipping. Derive an expression for tension in the cord T and coefficient of static friction μ between the wall and sphere, in terms of m, g, r, h, θ, and ϕ.



4. (15 pts) A solid cylinder with mass M is attached to a horizontal spring ($k = 3\ N/m$) and rolls without slipping.
   a. If the system is released from rest when the spring is stretched by 0.3 m, find the translational and rotational kinetic energy of the cylinder as it passes through the equilibrium position.
   b. Find the period of simple harmonic motion of the cylinder around the equilibrium point, in terms of M. Ignore the mass of the spring.



5. (10 pts) A uniform rope of length L and mass M hangs from a ceiling.
   a. Find the speed of the transverse wave on the rope as a function of y, the distance from the lower end of the rope.
   b. How long would it take for a transverse wave to travel the length of the rope?

6. (15 pts) A tuning fork is set into vibration above a vertical open tube filled with water. The water level is allowed to drop slowly. As it does so, the air in the tube above the water level is heard to resonate with the tuning fork when the distance from the tube opening to the water level is 0.125 m and again at 0.395 m, and there is no resonance that occurs between these two levels. What is the frequency of the tuning fork? The speed of sound in air is 343 m/s.

7. (10 pts) A rocket has a fuel canister of cross-sectional area A and an exit orifice of area A_0. Assume that $A_0 \ll A$.
   a. If the density of the gas is ρ, the pressure of the gas inside the rocket is P, and the atmospheric pressure just outside the orifice is P_0, then derive the emission speed of the propelling gases as they power the rocket. Assume that the density of the gas stays approximately constant and that there is no turbulence.
   b. Find the thrust force on the rocket due to the emitted gases.