University of California, Berkeley	**Last name:**	______________
Mechanical Engineering	**First name:**	______________
ME 106, Fluid Mechanics	**Student ID:**	______________
ODK/Midterm 2, Fall 2015	**Discussion:**	______________

Notes:

- You solution procedure should be legible and complete for full credit (use scratch paper as needed).

Question	Grade
1	
2	
3	
Total:	

1. Consider the unsteady, incompressible, 2D flow described by the velocity field

$$u = y^2 + t$$
$$v = x^2$$

   (a) Find the equation, $x(y)$ or $y(x)$, for the streamline passing through point $(0, 0)$ at time $t = 0$.

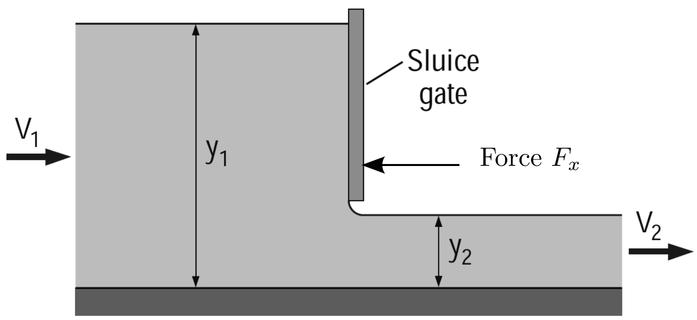
   (b) Find the equations, $x(t)$ and $y(t)$, describing a pathline originating at point $(0, 0)$ at time $t = 0$.

   (c) Calculate the acceleration of a fluid particle at point $(0, 0)$ at time $t = 0$.

2. A sluice gate of width b into the page controls the flow of water by raising or lowering a vertical plate. The water exerts a force $\mathbf{F}$ on the gate. Let ρ be the water density and other variables be as shown in the diagram. Disregarding the wall shear forces at the solid surfaces, and assuming steady, uniform flow:

   (a) Solve for the horizontal component of the force, F_x, the water imposes on the gate. Express answer in terms of $(\rho, y_1, y_2, b, g$ and $V_1)$

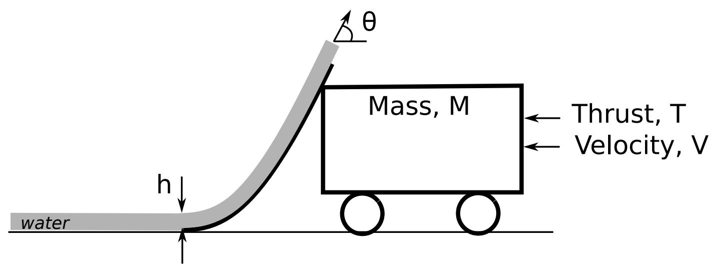
   (b) Based on the expression you derived above, derive an expression for y_2 when F_x is a maximum. Assume V_1 and y_1 remain constant.



3. A vehicle of mass M scoops stationary water of density ρ with depth h and width b into the page, creating an upward jet with angle θ. Assume the incoming and outgoing stream of water on the scoop have the same area. Neglect air drag, wheel friction and gravity effects.

   (a) Determine the thrust T to maintain a constant acceleration a in terms of the variables given.

   (b) Assume that the trust is removed ($T = 0$), hence the rocket decelerates from initial velocity V_0 at $t = 0$. Based on the expression you derived above, find the expression for the velocity $V(t)$ as a function of time (note: $a = \frac{dV}{dt}$).



Summary of Equations:

Chapter 4:

Equation for streamlines	$\frac{dy}{dx} = \frac{v}{u}$
Acceleration	$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$
Material derivative	$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V}\cdot\nabla)(\ )$
Streamwise and normal components of acceleration	$a_s = V\frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{\mathcal{R}}$
Reynolds transport theorem (restricted form)	$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$
Reynolds transport theorem (general form)	$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b\, d\forall + \int_{\text{cs}} \rho b\, \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$
Relative and absolute velocities	$\mathbf{V} = \mathbf{W} + \mathbf{V}_{\text{cv}}$

Chapter 5:

Conservation of mass	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = 0$
Mass flowrate	$\dot{m} = \rho Q = \rho A V$
Average velocity	$\overline{V} = \frac{\int_A \rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA}{\rho A}$
Steady flow mass conservation	$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0$
Moving control volume mass conservation	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = 0$
Deforming control volume mass conservation	$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\forall + \int_{\text{cs}} \rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = 0$
Force related to change in linear momentum	$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\\text{control volume}}}$
Moving control volume force related to change in linear momentum	$\int_{\text{cs}} \mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\\text{control volume}}}$
Vector addition of absolute and relative velocities	$\mathbf{V} = \mathbf{W} + \mathbf{U}$