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Physics H7B, Spring 2007 (*Xiaosheng Huang*)

<u>Midterm 1 Solutions</u>

1. a) For an ideal gas,

$$PV=nRT.$$

Solving for V,

$$V=nRT/P.$$

This is a constant pressure quasistatic process, P=const. Thus, we have

$$\mathrm{d}V/\mathrm{d}T=nR/P.$$

Finally,

$$\boxed{\beta=(1/V)(\mathrm{d}V/\mathrm{d}T)=nR/PV=1/T.}$$

b) At room temperature, $T=20^0\mathrm{C}=293$ K,

$$\boxed{\beta=1/293\mathrm{K}=3.4\times10^{-3}\ \mathrm{K}^{-1}.}$$

2) a) Again from ideal gas law, $PV=nRT$, we can solve for P,

$$P=nRT/V=Nk_BT/V=(N/V)k_BT$$

Now, $N/V=1\ \mathrm{cm}^{-3}=1/10^{-6}\ \mathrm{m}^3=10^6\ \mathrm{m}^{-3}$. This gives,

$$P=10^6\cdot1.38\times10^{-23}\cdot3000=4.14\times10^{-14}\mathrm{Pa}$$

In units of torr,

$$\boxed{P=4.14\times10^{-14}\mathrm{Pa}\cdot760\ \mathrm{torr}/1.01\times10^5\ \mathrm{Pa}=3.12\times10^{-16}\ \mathrm{torr}.}$$

(Compare this with the "ultra high vacuum" in a physics lab of 10^{-10} torr.)

b) The mean free path is given by,

$$\boxed{l_m=1/4\pi\sqrt{2}\,r^2(N/V)=1/4\pi\cdot1.41\cdot(5\times10^{-11})^2(10^6)=2.25\times10^{13}\mathrm{m}.}$$

3) a) Using Boltzmann's factor,

$$\boxed{P(E_1)/P(E_2)=e^{-0/\tau}/e^{-\varepsilon/\tau}=e^{\varepsilon/\tau}.}$$

b) To obtain the normalized probabilities,

$$P(E_1)=1/(1+e^{-\varepsilon/\tau})$$
$$P(E_2)=e^{-\varepsilon/\tau}/(1+e^{-\varepsilon/\tau}).$$

c) Now we can calculate the average energy,

$$<u>=0\cdot P(E_1)+\varepsilon\cdot P(E_2)$$

which leads to,

$$<u>=\varepsilon e^{-\varepsilon/\tau}/(1+e^{-\varepsilon/\tau}).$$

d) For N such system, the total energy is,

$$<U>=N<u>=N\varepsilon e^{-\varepsilon/\tau}/(1+e^{-\varepsilon/\tau}).$$

e) The constant volume specific heat is given by

$$C_V=(\partial U/\partial T)_V=N\,\partial/\partial T(\varepsilon/(1+e^{\varepsilon/\tau}))=-N\varepsilon(e^{\varepsilon/\tau}+1)^{-2}\cdot e^{\varepsilon/\tau}\cdot(-\varepsilon/k_BT^2)$$

and

$$C_V=(N\varepsilon^2/k_BT^2)(e^{\varepsilon/k_BT}/(e^{\varepsilon/k_BT}+1)^2)$$

4) a) Total initial internal energy is,

$$(3/2)nRT_1+(3/2)nRT_2=(3/2)nR(T_1+T_2),$$

for monatomic gas.

Total final energy is,

$$(3/2)(2n)RT_f.$$

From conservation of energy,

$$(3/2)nR(T_1+T_2)=(3/2)(2n)RT_f,$$

or,

$$T_f=(T_1+T_2)/2.$$

b) We choose the isochoric process since the volume doesn't change.

For box 1,

$$\Delta S_1=\int dS_1=\int đQ/T_1.$$

For the isochoric process, $dU=đQ-đW=đQ=C_VndT$. Thus,

$$\Delta S_1 = \int_{T_1}^{T_f} \frac{C_V n_1 dT}{T} = nC_V \ln(T_f/T_1) = nC_V \ln((T_1+T_2)/2T_1)$$

Similarly, for box 2,

$$\Delta S_2 = nC_V \ln((T_1+T_2)/2T_2)$$

$$\Delta S_{tot} = \Delta S_1 + \Delta S_2 = nC_V \ln((T_1+T_2)^2/4T_1T_2)$$

Now, we know

$$(T_1-T_2)^2 \geq 0 \qquad (1),$$

but

$$(T_1-T_2)^2 = T_1^2 + T_2^2 - 2T_1T_2 = (T_1+T_2)^2 - 4T_1T_2 \qquad (2).$$

Combining (1) and (2), we get

$$(T_1+T_2)^2 - 4T_1T_2 \geq 0 \text{ or } (T_1+T_2)^2/4T_1T_2 \geq 1$$

Thus,

$$\Delta S_{tot} \geq 0$$

c) We know,

$$\sigma(N, U) = \ln g(N, U) = (3/2)N\ln(KU) = (3/2)N(\ln K + \ln U)$$

For an ideal gas,

$$U = (3/2)N\tau$$

Thus we have,

$$\sigma = (3/2)(N\ln K + N\ln((3/2)(N\tau)))$$

The entropy change then is,

$$\Delta\sigma = (3/2)(N\ln K + N\ln N + \ln((3/2)\tau_f)) - (3/2)(N\ln K + N\ln N + \ln((3/2)\tau_i))$$

For box 1,

$$\Delta\sigma = (3/2)N\ln(\tau_f/\tau_i)$$

or,

$$\Delta\sigma = (3/2)N\ln((T_1+T_2)/2T_1)$$

Thus,

$$\Delta S = k_B \Delta\sigma = (3/2) N k_B \ln((T_1+T_2)/2T_1),$$

which agrees with part (b).