Math 54, Spring 2015 | Professor Mariusz Wodzicki

Discussion Section # | Your Name and S.I.D.

Midterm 1

March 13, 2015

Problem	1	2	3	4	5	Total
Score						

1. Use the cofactor expansion to calculate the determinant of the matrix

$$\begin{bmatrix} a & 0 & 0 & 0 & 0 \\ b & b & 0 & 0 & 0 \\ c & c & c & c-1 & c-1 \\ 0 & d & c+1 & c & c+1 \\ 0 & 0 & 0 & 0 & d \end{bmatrix}$$

2. Consider two bases $\mathcal{B} := \{1, t, t^2, t^3\}$ and $\mathcal{C} := \{1, t+1, (t+1)^2, (t+1)^3\}$ in the vector space $\mathbf{P}_3$ of polynomials of degree ≤ 3. Find the change-of-basis matrix $\mathcal{C} \overset{P}{\leftarrow} \mathcal{B}$.

3. Let $T\colon V \to V$ be a linear transformation with the characteristic polynomial

$$-2+3\lambda^2-\lambda^3.$$

Answer the following questions:

a) What is the value of $\dim V$?

b) What is the value of $\det T$?

c) What is the number of distinct real eigenvalues? How many eigenvalues are positive?

d) Is T diagonalizable? If you say it is not diagonalizable, explain the reason. Do likewise if you say it is diagonalizable.

4. If the rank of a 8×7 matrix A is 5 what is the dimension of the solution space $A\mathbf{x} = 0$? Explain your answer.

5. Let

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix},$$

be linearly independent vectors in $\mathbf{R}^3$. Let $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation that cyclically permutes the above vectors,

$$T(\mathbf{a}) = \mathbf{b}, \qquad T(\mathbf{b}) = \mathbf{c} \qquad \text{and} \qquad T(\mathbf{c}) = \mathbf{a}.$$

Find the eigenvalues and determine all eigenvectors for T. Is T diagonalizable? Explain your answer.