UC Berkeley – Computer Science
CS61BL: Data Structures

Final, Summer 2016

This test has 11 questions worth a total of 65 (with 5 as extra credit) points. The exam is closed book, except that you are allowed to use two double-sided pages of notes as a cheat sheet. No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided.

Write the statement out below in the blank provided and sign. You may do this before the exam begins. **Any plagiarism, no matter how minor, will result in points deducted from your exam.**

"I have neither given nor received any assistance during the taking of this exam."

_____

_____

Signature: _____

Write your name and student ID on the front page. Write the names of your neighbors. Write and sign the given statement. Once the exam has started, write your login in the corner of every page.

Name: _____ **Your Login:** cs61bl-_____

SID: _____ **Name of person to left: _____**

TA: _____ **Name of person to right: _____**

Notes:

- **All graphs are simple graphs: two vertices can only have one edge between them, and there are no self-edges.**
- There may be partial credit for incomplete answers. Write as much of the solution as you can, but bear in mind that we may deduct points if your answers are much more complicated than necessary.
- There are a lot of problems on this exam. Work through the ones with which you are comfortable first. Do not get overly captivated by interesting design issues or complex corner cases you're not sure about.
- Not all information provided in a problem may be useful.
- Unless otherwise stated, you can use any standard library classes & methods, and can assume imports happen automatically.
- Unless otherwise stated, all given code on this exam should compile. All code has been compiled and executed before printing, but in the unlikely event that we do happen to catch any bugs during the exam, we'll announce a fix. Unless we specifically give you the option, the correct answer is not 'does not compile.'

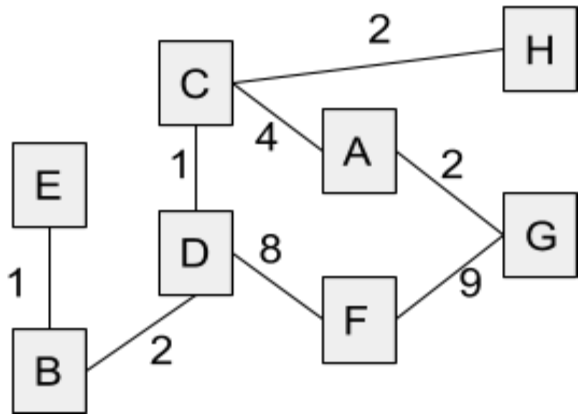
Optional. Mark along the line to show your feelings on the spectrum between ☹ and 😂.

Before exam: [☹ _____ 😂]

After exam: [☹ _____ 😂]

1. Montague's (8 pts)

a. For the graph below, write the order in which vertices are visited using the specified algorithm. Each node's neighbors are given in alphabetical order. The starting point, vertex A, is provided for you.

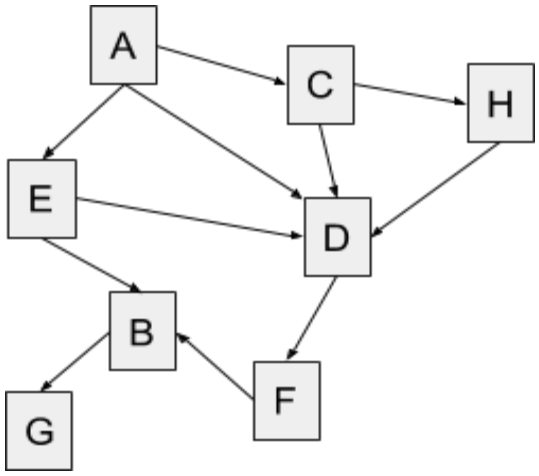


DFS: A ____ ____ ____ ____ ____ ____ ____

BFS: A ____ ____ ____ ____ ____ ____ ____

Dijkstra's: A ____ ____ ____ ____ ____ ____ ____

b. Given the following graph,

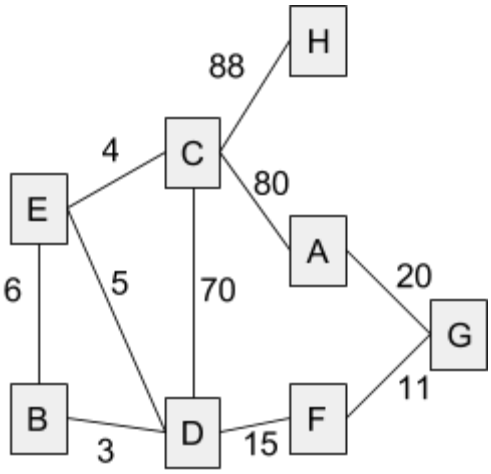


What is a valid topologically sorted ordering of the vertices?

____ ____ ____ ____ ____ ____ ____ ____

How many possible orderings are there? _____

Parts **c**-**e** refer to the following weighted undirected graph to the right.



c. Draw the intermediate state of the graph during execution of Kruskal's Algorithm after four edges have been added:

d. Draw a **tree representation** of the union-find (disjoin sets) data structure, **without path compression**, at the same point in the algorithm:

e. Now suppose the algorithm has reached completion, but designers want to secretly add another edge to the graph of integer weight `w`. For each of the following conditions, circle whether adding an edge of that weight to the graph will, might, or will not change the edges in a MST:

If `w < 4`, a MST **[will / might / will not]** change.

If `5 < w < 7`, a MST **[will / might / will not]** change.

If `20 < w`, a MST **[will / might / will not]** change.

If `90 < w`, a MST **[will / might / will not]** change.

2. Kimchi Garden (6 pts)

For any **weighted, undirected** graph, determine whether each statement is true (T) or false (F) and circle your choice. If you choose false, provide a counterexample in the space below the statement.

a. Adding 1 to each edge weight will not change any MSTs. **T** **F**

b. Adding 1 to each edge weight will not change the shortest paths between vertices. **T** **F**

c. If a graph of V vertices has more than |V| - 1 edges, and there is a unique edge of highest weight, it cannot be a part of *any* MST. **T** **F**

d. The shortest path between two nodes s and t must be a part of *some* MST. **T** **F**

e. If we could sort a list in O(1) time, the runtime of Kruskal's would asymptotically improve. **T** **F**

f. If there is a path between all pairs of vertices in a directed graph, there must be a cycle in the graph. **T** **F**

3. Tamon Tea (2 pts)

Recall the class definition of `IntNode` as defined below:

```
public class IntNode {
    int item;
    IntNode next;
    public IntNode(int item, IntNode next) {
        this.item = item;
        this.next = next;
    }
}
```

Write the `breakTheLoop` method below, which takes in an `IntNode`. You are guaranteed that the list this node belongs to is **circular** (the "last" element of the list points to the "first"), it is in **sorted increasing** order, and all its items are **unique**. However, the node passed in to `breakTheLoop` can point to any node in the circular list.

`breakTheLoop` should destructively turn the list into a non-circular list where the first node contains the lowest number and return the first node. You may iterate through the circular list at most once. You may not need all the lines below.

```
public static IntNode breakTheLoop(IntNode n) {

    assert n != null;

    ____________________________________________________________

    while (____________________________________________________________) {

        ____________________________________________________________

        ____________________________________________________________

    }

    IntNode toReturn = _____________________________;

    ____________________________________________________________

    return toReturn;

}
```

4. Jasmine Thai Lunch Special (5 pts)

a. Provide the best case and worst case runtimes in theta notation in terms of N for the following operations and data structures. Assume `N` to be the number of nodes in the tree. Additionally, each node correctly maintains the size of the subtree rooted at it.

Operations	BST	Red-Black tree
`// Returns true if the object is in the tree` `boolean contains(T o);`	Best: Θ( ) Worst: Θ( )	Best: Θ( ) Worst: Θ( )
`// Inserts the given object.` `void insert(T o);`	Best: Θ( ) Worst: Θ( )	Best: Θ( ) Worst: Θ( )
`// Returns the ith smallest object in the tree.` `T getElement(int i);`	Best: Θ( ) Worst: Θ( )	Best: Θ( ) Worst: Θ( )

b. Professor Sarallahan Kyao at Harbuvard University decided to use a self-balancing binary search tree. However, the code became somehow corrupt and you, as a faithful assistant, have the job of fixing it.

At each node, we keep an `int` variable called `height`, which keeps track of the height of the subtree rooted at the node. A leaf node has a height of 0, and each non-leaf node has a height of `1 + max(left.height, right.height)`. Professor Kyao demands that for each node, the left child's height and right child's height differ by no more than 1. That is, `Math.abs(left.height - right.height) <= 1`. Thus, whenever one of our left or right subtrees is not balanced, or their height differs by more than 1, we should rebalance.

Fill in the blanks below so that the code correctly calls `rebalance()` as needed.

```
class WhatTree {
    Node root;
    static class Node {
        int item, height;
        Node left, right;


        Node(int item, int height) {
            this.item = item;
            this.height = height;
        }
    }

    public void insert(int item) {
        root = insert(root, item);
    }

    // Continued on next page...
```

```
    private static Node insert(Node node, int item) {
        if (node == null) {
            return ____________________________________________________________;
        }
        if (node.item == item) {
            return node;
        } else if (node.item < item) {
            ____________________________________________________________;
        } else {
            ____________________________________________________________;
        }
        node.height = findHeight(node);

        if (____________________________________________________________) {
            node = rebalance(node);
        }
        return node;
    }

    private static boolean isBalanced(Node n) {
        if (n == null) return true;
        if (Math.abs(_____________________________________________) > _______) {
            return _________________;
        } else if (!isBalanced(_________________) || !isBalanced(_________________) {
            return _________________;
        }
        return _________________;
    }

    // Finds the height of a node.
    private static int findHeight(Node n) { ... }

    // Assume this works properly
    private static boolean rebalance(Node n) { ... }

}
```

5. SF Soup Company (4 pts)

a. Consider a `HashMap<Integer, String>` with an underlying array of size 5. Draw the resulting structure after the following operations.

```
put(3, "monument");
put(8, "shrine");
put(3, "worker");
put(5, "granary");
```

b. Two instances of a class are **deep-equals** if all its fields are `.equals()`. Consider the `FastMap`, a variant of a `HashMap` that avoids collisions. A `FastMap` requires the following for every key K inserted into it: (1) Instances of K have a unique final int id field, which is set upon construction to the number of unique instances of K created so far. (2) The hashCode() method for K returns id. (3) The equals(Object o) method for K returns whether the two objects are deep equals. (4) Once a FastMap is instantiated, no more instances of K can be constructed.

The `FastMap`'s underlying array size is initialized to be the number of instances created (the highest `id`) and a `FastMap` does not resize.

What is the worst case runtime of `put`, `get`, and `remove` (they are the same) on a `FastMap`?

Runtime: _____

If the `FastMap` does indeed prevent collisions, justify your answer. Otherwise, give a counterexample.

6. Momo Masala (7 pts)

a. For each of the following scenarios below, fill in the blank with the letter corresponding to the sorting algorithm that would perform the best, with respect to **total runtime**, on sorting a **large array** (unless otherwise specified). Then provide the tight **asymptotic runtime** of the sort you chose, on the array, in Big-O notation, where N is the length of the array (for quicksort, provide the average/majority case runtime).

A. Insertion Sort B. Mergesort C. In-Place Quicksort D. Radix sort E. Bubble Sort

		Letter	Runtime
1.	Java `ints`.	_____	O(_____)
2.	Floating point numbers of any length (e.g. 2.5, 3.1415)	_____	O(_____)
3.	Comparable objects.	_____	O(_____)
4.	Comparable objects, given that there are O(N) inversions.	_____	O(_____)
5.	Comparable objects, given that there are $O(N^{1.5})$ inversions.	_____	O(_____)
6.	Strings of length $O(\log(N)^2)$.	_____	O(_____)
7.	Java `ints`, and the array length is less than 40.	_____	O(_____)
8.	Linked List of Comparable objects. .	_____	O(_____)

b. Suppose you are given a list of N objects, comparable in O(1) time. Based on the data set you are working with, you know the objects can only be in one of $O(4^N)$ possible permutations. Give an asymptotic lower bound on the running time[1] of sorting this list. Briefly justify your answer.

Lower bound: _____
Justification:

What does this imply about the space complexity of a sorting algorithm that has the runtime of the lower bound? Provide an asymptotic upper bound of the space complexity. Justify your answer.

Upper bound: _____
Justification:

Now suppose you know the objects can only be in $O(N^2)$ possible permutations. Again, provide an asymptotic lower bound on the running time of sorting this list. Justify your answer.

Lower bound: _____
Justification:

[1] Disclaimer: in the decision tree model of computing. Don't worry about it.

7. Snack Shack: The Pork Belly Cubano (8 pts + 5 EC)

Provide a brief description of how to solve each of the following graph problems efficiently. Provide a worst-case runtime bound in Big-Θ notation in terms of V and E. Let WUG mean weighted, undirected graph.

a. Find a path from node s to node t in a strongly connected, directed graph.
Description:

Runtime: _____

b. On a connected WUG, find the shortest path from s to t that goes through a given edge `e=(u,v)`.
Description:

Runtime: _____

c. Determine if a cycle exists in a directed graph.
Description:

Runtime: _____

d. Given a connected WUG, find the maximum spanning tree, the spanning tree of maximal weight.
Description:

Runtime: _____

e. (Extra Credit, 2 pts) Find the longest path between any two nodes in a weighted, directed, acyclic graph.
Description:

Runtime: _____

f. (Extra Credit, 3 pts) Given a weighted, undirected graph where the weights are bounded by k, give an algorithm and data structure to find the shortest s-t path in O((|V|+|E|) log(k)) time.
Description:

8. Little Gem Belgian Waffles (5 pts)

a. Less than a hundred years after returning to the Undying Lands, Elrond has found a new hobby: forging legendary water bottles to be given away at Valar career fairs. He is an ardent fan of Java, so he represents each of his creations as a `WaterBottle implements Comparable<WaterBottle>` and ranks them on a large number of quality factors. This way, when he sorts them, the water bottles are ranked from worst to best.

He wants to send the highest `k` quality water bottles of his `n` water bottles back to his daughter Arwen, where `k` is still undecided. He doesn't want to sort the entire list of water bottles, because that would be too slow. Help him implement a function that runs in **`Θ(n log k)`** time that will find the `k` highest quality water bottles in descending order.

Recall that `PriorityQueue`, `ArrayDeque`, and `LinkedList` implement `Queue`. `ArrayList` and `LinkedList` implement `List`. *You may not need all lines.*

```
public static List<WaterBottle> topK(List<WaterBottle> all, int k) {
    List<WaterBottle> ret = ____________________________________________;
    Queue<WaterBottle> q  = ____________________________________________;
    for (_____________________________________________) {
        q.add(_________________);
        ____________________________________________
        ____________________________________________
        ____________________________________________
    }
    while (_____________________________________________) {
        ____________________________________________
        ____________________________________________
    }
    return ret;
}
```

Give a brief justification of the runtime:

_____

_____

9. Kamado Sushi (4 pts)

The quicksort algorithm can be modified for finding the `k`th smallest element in an array. This is called the quickselect algorithm as briefly discussed in lab, and finds the item at sorted index `j = k - 1`. The partition step is the same as quicksort, and it only differs in the recursive call, recursing on the partition that contains the `k`th smallest element.

a. Using the first element of the list as the pivot, show the way the list is partitioned at each step when `quickselect(j = 5)` is called, trying to find the sixth smallest element. Circle the partition that is recursed on.

```
42   93   50   39   81   94   23   28   95   89
```

Left Partition	Pivot Partition	Right Partition	j
	`42`		5

b. What is the best, worst, and average case runtime of quickselect?

Best: _____

Worst: _____

Average: _____

c. Quickselect can be used to find the median of an array. What if, instead of selecting a pivot at random when quicksorting, instead we use quickselect to find the median and use that as the pivot? What is the worst, best, and average case runtime of quicksort now?

Best: _____

Worst: _____

Average: _____

10. Chengdu Restaurant (4 pts)

For the below regex problems, assume these are **non-java** strings, so double escaping is not necessary.
a. Consider the regex `"[hello]\w+rld"`. Circle all strings that the regex fully matches.

`"helloworld"` `"hworld"` `"smallworld"` `"hellowrld"` `"otherworld"`

b. Consider the regex `"[a-d0-9]+\\[^\d]{2,5}"`. Circle all strings that the regex fully matches.

`"d00d+42"` `"61b\party"` `"aa61bb+cc"` `"000+ooo"` `"9a9\\+hue"` `"4242+meaning"`

For parts **c**-**e**, write regular expressions that will fulfill the conditions below. Be sure to escape special characters.

c. Match any occurrences of a string that has an "A" at the beginning and end, and greedily, any positive number of "B" in between.
Fully matched inputs: `"ABA"`, `"ABBBBBBA"`
Non-matching inputs: `"AAAABBBB"`, `"AAAA"`, `""`, `"BBBABBB"`, `"AABABABABABA"`

Regex: _____

d. Match a valid alphanumeric lowercase email address.
Fully matched inputs: `"alan42@cs61bl.io"`, `"party@gmail.com"`
Non-matching inputs: `"dun@goofed"`, `"CAPITALS@stop.shouting"`, `"n.onalph_anum@r1c.com"`

Regex: _____

Parrot Party (This parrot speaks to me on a transcendental level)

This is a designated ExamFunOnlyZone™©. Draw or write whatever you want!

11. Ellenos Real Greek Yogurt (7 pts)

a. We can implement a `PriorityQueue` an array-based binary heap. Consider a heap where the keys are integers, and the priority is the key itself. Write out the values of the array representation after the following calls:

```
Starting array:           [ X , 0   , 6   , 1   , 9   , 7   , 4   , 13  , 17  , 12  ]
removeMin():              [ X , ____, ____, ____, ____, ____, ____, ____, ____]
add(3):                   [ X , ____, ____, ____, ____, ____, ____, ____, ____, ____]
changePriority(17, 2):    [ X , ____, ____, ____, ____, ____, ____, ____, ____, ____]
```

b. Recall that `changePriority` on a `PriorityQueue` takes linear time. This means that we cannot use `changePriority` as given in Dijkstra's algorithm, otherwise the runtime will not be satisfactory. Augment the `ArrayHeap` that you wrote in lab18 such that `changePriority` takes O(log n) time. You may not need all lines.

```
public class ArrayHeap<T> {
    private ArrayList<Node> contents;
    /** Put any additional fields here and initialize them in the constructor. */



    public ArrayHeap() {
        contents = new ArrayList<>();
        contents.add(null);


    }
    private class Node {
        T item; double priority;
        /** Put any additional fields here and initialize them in the constructor. */



        private Node(_____________________________________________________) {
            this.item = item;
            this.priority = priority;


        }
    }

    /** Bubbles up the node currently at the given index. */
    private void bubbleUp(int index) { ... }

    /** Bubbles down the node currently at the given index. */
    private void bubbleDown(int index) { ... }

    // Continued on next page...
```

```
    /** Always called whenever nodes are swapped by bubbleUp and bubbleDown */
    private void swap(int index1, int index2) {
        Node node1 = contents.get(index1);
        Node node2 = contents.get(index2);
        contents.set(index1, node2);
        contents.set(index2, node1);

        ____________________________________________________________________

        ____________________________________________________________________

    }

    /** Inserts an item with the given priority value. Same as enqueue, or offer. */
    public void insert(T item, double priority) {
        Node n = new Node(_____________________________________________________);
        contents.add(n);
        bubbleUp(contents.size() - 1);

        ____________________________________________________________
    }

    /** Returns and removes the Node with the smallest priority value. */
    public Node removeMin() {
        swap(1, contents.size() - 1);
        Node results = contents.remove(contents.size() - 1);
        bubbleDown(1);

        ____________________________________________________________
        return results;
    }

    /** Changes the node in this heap with the given item to have the given
     * priority. You can assume the heap will not have two nodes with the same
     * item. You can assume item is indeed in the heap. */
    public void changePriority(T item, double priority) {

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

        ____________________________________________________________

    }
}
```