

**Instructions: *Read them carefully!***

*The exam begins at 3:10pm and ends at 6:00pm. You must turn your exam in when time is announced or risk not having it accepted.*

*Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.*

**Write legibly.** *If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.*

**Be clear and concise.** *The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points than short ones do because there are more opportunities to mark something wrong.*

*You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student's exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.*

**Do not ask questions during the exam.** *Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.*

***I have read these instructions, I understand them, and I will follow them.***

Your Name: \_\_\_\_\_

Your Signature: \_\_\_\_\_

Date: \_\_\_\_\_

Student ID: \_\_\_\_\_

Class account: \_\_\_\_\_

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1. Please fill in each of the blanks with an appropriate answer.

*2 points each blank*

If the singular value decomposition of a matrix is  $A = QSP^T$ , then the psuedo-inverse of the matrix is given by  $A^{-P} =$  \_\_\_\_\_.

The cross-product of the tangent vectors of a parametric surface generally can be used to comput-  
ed the \_\_\_\_\_.

When representing \_\_\_\_\_ in 3D using homogenized coordinates, the fourth  
coordinate (i.e. "w") will be **non-zero**.

The \_\_\_\_\_ rendering method assumes that all materials in a scene are dif-  
fuse.

The \_\_\_\_\_ rendering method computes a view-independent solution..

*Irradiance (a.k.a. radiant exitance)* is measured in units of \_\_\_\_\_.

Catmull-Clark subdivision surfaces are a generalization of uniform, cubic, tensor-product  
\_\_\_\_\_ surfaces.

A B-spline curve is always enclosed by the \_\_\_\_\_ of its control  
points.

In Catmull-Clark subdivision, the number of new extraordinary points introduced on the **first** round  
of subdivision will be equal to the number of \_\_\_\_\_ .

The \_\_\_\_\_ of an orthonormal matrix is equal to its *transpose*.

\_\_\_\_\_ encode 3D rotations as 3D points inside a ball of radius  $\pi$  *radians*.

The special case of a perspective camera that is infinitely far away from a scene is termed a(n)  
\_\_\_\_\_ camera.

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A texture mapping method called \_\_\_\_\_ is used to change the apparent shape of an object during shading by perturbing the surface normals.

NURBS are non-uniform \_\_\_\_\_ B-Splines that use homogeneous coordinates for control points.

Steradians are the dimensionless units used to measure \_\_\_\_\_.

Finding the intersection of a ray with a sphere requires solving a \_\_\_\_\_ equation.

Cloth simulations using forward Euler integration typically become \_\_\_\_\_ unless a large amount of damping is used.

The dynamic range of the human eye is much \_\_\_\_\_ than the dynamic range of a typical LCD television set.

If a spring with length  $l$  has stiffness coefficient  $k$ , then a pair of springs in serial with length  $l/2$  should have stiffness \_\_\_\_\_ if they are to replicate the behavior of the original spring.

**2. Answer the following questions with True (T) or False (F)**

*2 points each*

\_\_\_\_\_ Light transport can be modeled reasonably well using a collection of particles attached by radiance-links.

\_\_\_\_\_ The Jacobian of a valid kinematic system will sometimes be invertible, depending on the system's configuration.

\_\_\_\_\_ Shiny metal surfaces typically have bright metal-colored specularities.

\_\_\_\_\_ In a pool of cloudy water the radiance along a straight line would fall off exponentially.

\_\_\_\_\_ The rods in the human eye have a spectral response function that peaks somewhere between the short and medium cones' responses.

\_\_\_\_\_ Under linear perspective projection, squares always appear as rectangles unless the projection is degenerate.

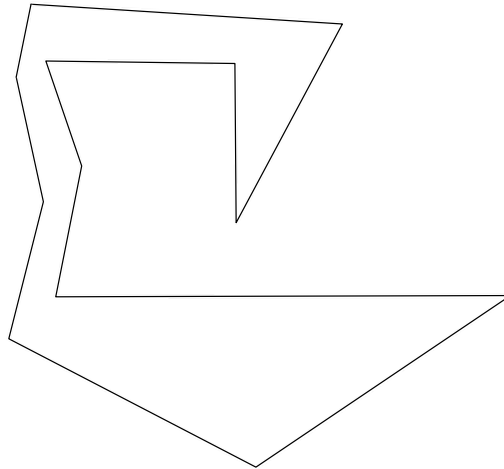
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- \_\_\_\_\_ Under linear perspective projection, any triangle always will appear to have at least one angle less than 90 degrees.
  - \_\_\_\_\_ Quaternions represent rotations as points in 3D space on the surface of a hyper-sphere.
  - \_\_\_\_\_ Shining an ultraviolet light on scorpions makes them secrete an acidic toxin that glows bright yellow.
  - \_\_\_\_\_ The force exerted by a linear-strength spring with non-zero rest length is given by a function that is polynomial in terms of the endpoint locations.
  - \_\_\_\_\_ Cubic Bezier curves will be  $C^2$  across segment boundaries.
  - \_\_\_\_\_ Surface texture is generated by non-normalized permutation maps.
  - \_\_\_\_\_ In a valid kinematic skeleton, every parent body must have exactly one child body.
  - \_\_\_\_\_ A rotation matrix always has determinant that is less than zero.
  - \_\_\_\_\_ Pasteurized coordinates facilitate representing perspective and translation using matrices.
  - \_\_\_\_\_ Ambient occlusion tends to enhance the appearance of surface detail.
  - \_\_\_\_\_ The sky is blue because water vapor scatters light in the long part of the spectrum.
  - \_\_\_\_\_ In some women tetrachromacy is caused by a mutation in the coding for the cones.
  - \_\_\_\_\_ In a rectilinear spring mesh, adding "jump" springs will make the mesh rigid.
  - \_\_\_\_\_ Motion graphs used for animating human figures should never contain cycles.
  - \_\_\_\_\_ Given several recorded human motion sequences that appear natural, motions created by blending them will also appear natural and human-like because human perception is tri-linear.
  - \_\_\_\_\_ Planar inverse kinematics problems will typically have simple closed-form solutions.
  - \_\_\_\_\_ A ball joint represented with an exponential map has four degrees of freedom.
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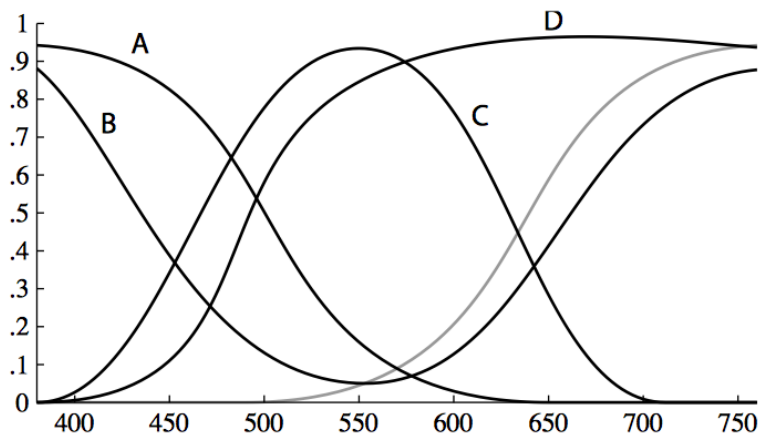
- \_\_\_\_\_  $C^1$  continuity does not always imply  $G^1$  continuity
  - \_\_\_\_\_  $G^1$  continuity does not always imply  $C^1$  continuity
  - \_\_\_\_\_ The Bezier basis functions are affine invariant.
  - \_\_\_\_\_ The fully implicit version of Euler's method (a.k.a. backward Euler) is generally stable.
  - \_\_\_\_\_ Some motion capture systems use magnetic fields to determine the location and orientation of tracker objects.
  - \_\_\_\_\_ Vector-based image representations use round pixels to avoid aliasing.
  - \_\_\_\_\_ Non-zero winding number and parity testing will always produce the same result for polygons with self-intersecting boundaries.
  - \_\_\_\_\_ Particle systems simulate objects such as waterfalls by modeling the detailed interactions between individual molecules of water.
  - \_\_\_\_\_ The result of applying subdivision to a cubic curve is one quadratic (lower order) curve.
  - \_\_\_\_\_ Catmull-Clark subdivision can be accelerated using BSP-Trees or K-D Trees.
  - \_\_\_\_\_ The long cones in the human eye only sense red light.
  - \_\_\_\_\_ In a bounding-box tree, the bounding-box stored at node in the tree must encompass the box of its parent node.
  - \_\_\_\_\_ The B-spline basis functions have finite support.
  - \_\_\_\_\_ Texture-mapping will not change an object's surface color.
  - \_\_\_\_\_ Polynomial basis functions can be used to build perfect circles.
  - \_\_\_\_\_ Turning your final assignment in late will result in a zero on the assignment!
  - \_\_\_\_\_ A rotation matrix always has determinant of +/-  $\pi$ .
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3. Draw the convex hull of the shape shown below.

2 points



4. Write the common English name for each of the color matching each of the following spectral density curves. For example, the unlabeled curve would be red. 8 points



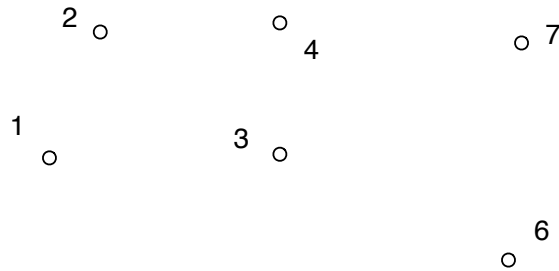
A. \_\_\_\_\_

B. \_\_\_\_\_

C. \_\_\_\_\_

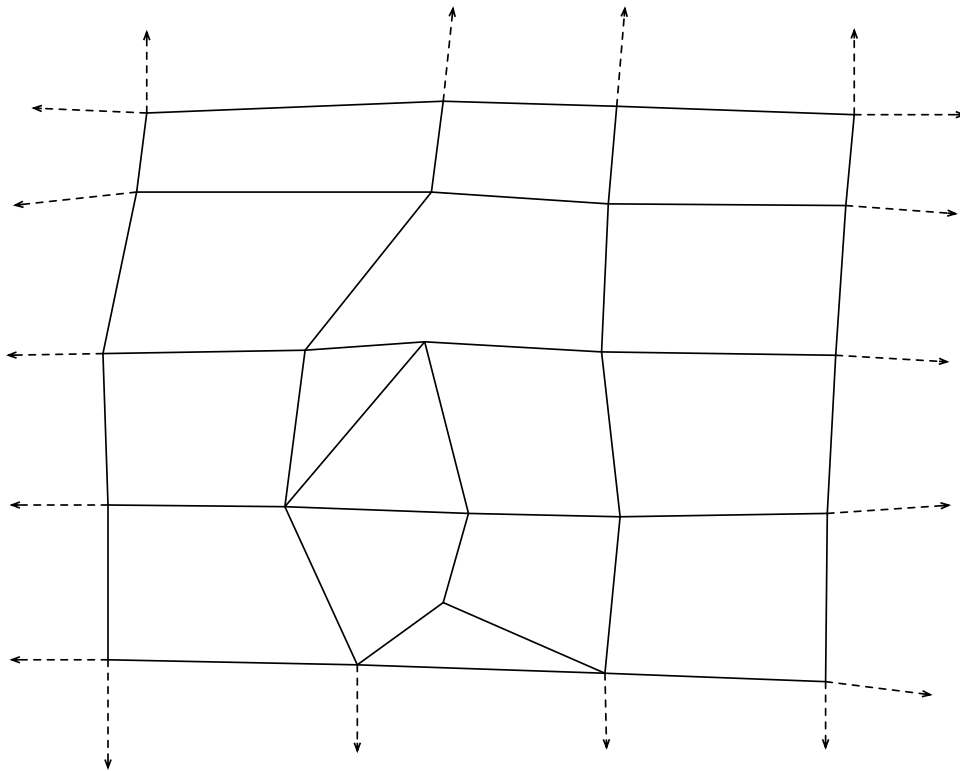
D. \_\_\_\_\_

5. The diagram below shows control points for a curve made by joining two cubic Bezier segments. However control point #5 has been removed. Indicate a location where #5 may be placed to achieve  $C^1$  continuity and draw the curve that would result. Also draw a line where #5 may be placed to achieve  $G^1$  continuity. *Make sure your diagram is clear and geometrically reasonable.* *9 points*



6. Name a phenomenon that can be modeled easily using the radiosity method but that cannot be modeled with a basic ray-tracing algorithm. Give an example. *3 points*
7. Given a rotation matrix, how would you determine the axis that it rotates around? *3 points*
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8. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. Note: I am only interested in the topology of your answer, but make sure your diagram is clear. 7 points



9. Below are two 4x4 homogenized transformation matrices. Describe what each of them will do. 4 points

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1/2 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

The first one will: \_\_\_\_\_

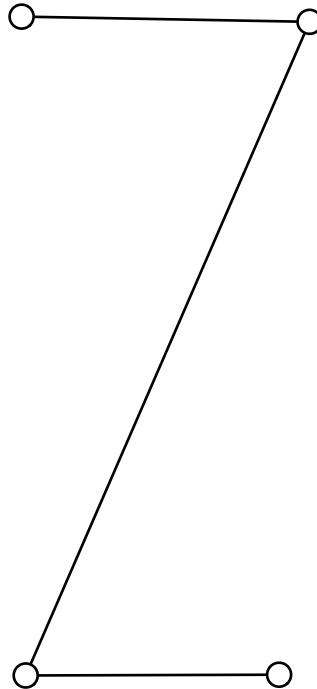
The second one will: \_\_\_\_\_



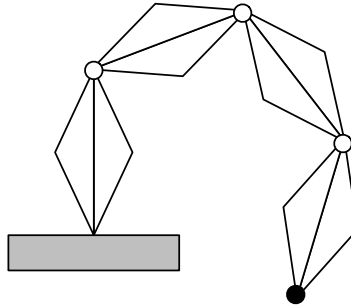
10. Write out an implicit equation for a 2D ellipse where the long axis is the X axis with radius 7 and the short axis is the Y axis with radius 3. *8 points*

11. Write out a parametric equation for a plane in 3D that is parallel to the X-Y Plane and passes through some point P. *3 points*

12. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljau's algorithm can be used to subdivide the curve into two equal halves. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the beginning, middle, and end of each segment. *5 points*



13. Consider this diagram showing a four-joint arm in 2D where each joint is a simple pin joint and the base is fixed in space.



If we are solving an IK problem to place the tip of the arm (the black dot) at a particular location, what is the size of the Jacobian matrix we will be working with? 3 points

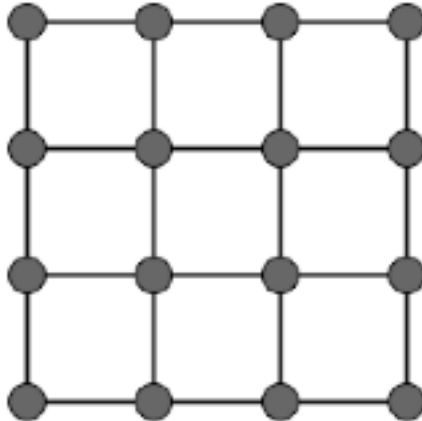
Draw any one configuration of the arm where two columns of the Jacobian will be parallel vectors. In the drawing *clearly* show the direction of the parallel vectors. 7 points

When will the this system's Jacobian be fully invertible? 1 point

14. When rendering a scene with a *photon-mapping* method, what part of the solution must be recomputed when the viewer moves? 4 points

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15. On the diagram below, draw the springs that should be added to provide some *unbiased* resistance to in-plane shearing. *4 points*



16. What limits the size of the capture region that can be used with a magnetic motion capture system? *2 points*

17. Explain the relation between The Rendering Equation and the ray-tracing algorithm. *8 points*
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**EXTRA CREDIT***10 points***Given a sphere and plane:**

**Sphere:**  $\|\mathbf{x} - \mathbf{c}\| - r^2 = 0$

**Plane:**  $\mathbf{x}(u, v) = u\mathbf{v}_1 + v\mathbf{v}_2 + \mathbf{p}$

**Write out an explicit parametric equation that produces the circle where the sphere and plane intersect. Also indicate when this circle is undefined. You may assume that  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are both of unit length and mutually orthogonal.**

Your answer must be neat and clear. No points will be awarded for imprecise or messy answers. Your answer should be in the form of a simple explicit equation that you have drawn a box around. Do not attempt this question until you have completed the rest of the exam! There will be no partial credit for this question.

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