EECS 120
Midterm 1
Wed. Oct. 26, 2016: 1610 - 1800 pm

Name:____________________________ person left:____________________________
SID:______________________ person right:____________________________

For statistical purposes only:
Circle courses you have taken EE20 EE16B neither

- Closed book. One 8.5x11 inch page double sided formula sheet. No calculators.
- There are 5 problems worth 100 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	22	
2	25	
3	19	
4	24	
5	10	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Tables for reference:

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

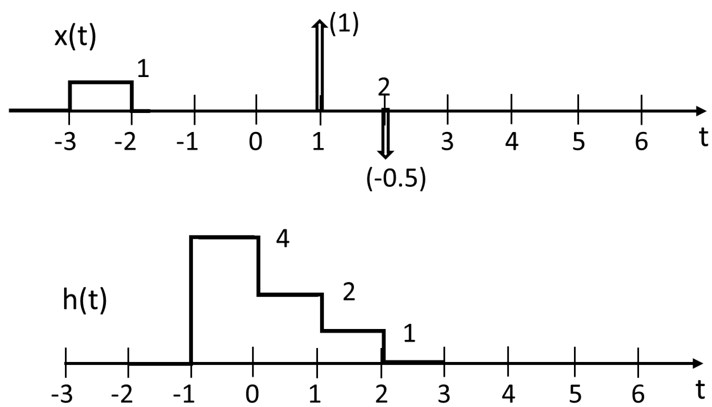
$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 LTI Properties (22 pts)

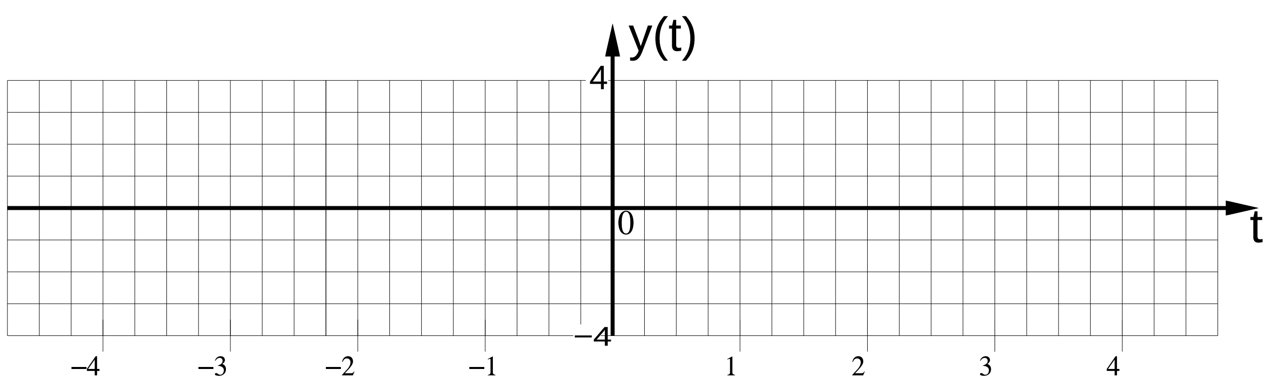
[16 pts] a. Classify the following systems, with input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).
Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$

System	Causal	Linear	Time-invariant	BIBO
i. $y(t) = x(t) * \Pi(t)$				
ii. $y(t) = x(t) \cdot [\Sigma_{n=-\infty}^{\infty} \delta(t-2n) * \Pi(t)]$				
iii. $y[n] = x[n] \cdot y[n-2] + u[n-2]$				
iv. $y(t) = \int_{-1}^{1} x(\tau)\Pi(t-\tau)d\tau$				

[6 pts] b. An LTI system has input $x(t)$ and impulse response $h(t)$ as shown below:

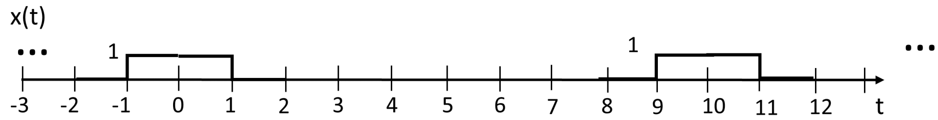


Sketch the output $y(t)$ on the grid below, noting key times and amplitudes.



Problem 2 Fourier Series (25 pts)

You are given a periodic function $x(t)$ as shown, where the shape is a rectangular pulse of height 1 and width 2, centered at $t = 0$:

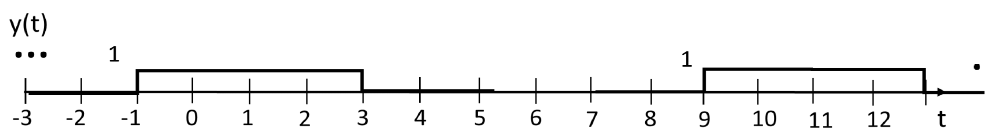


Note that $x(t)$ can be represented by a Fourier Series: $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$.

[1 pts] a. What is the fundamental frequency $\omega_o =$ __________

[8 pts] b. Find a_k using the Fourier Series analysis equation. $a_k =$ __________________________ (Show your calculation).

Given a new signal $y(t)$ as shown:



Periodic function $y(t)$ can be represented by a Fourier Series: $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_o t}$

[6 pts] c. Show how to find b_k in terms of a_k (answer should be left in terms of a_k) .

Problem 2, continued.

[4 pts] d. If $y(t) = x(t) * h(t)$,
(with $y(t)$ from part c and $x(t)$ from part a,b) find $h(t) =$: ________________________

A signal $c(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$ is passed through an LTI filter $g(t)$ with impulse response:

$$g(t) = \frac{\pi}{3} e^{\frac{-\pi}{3}t} u(t)$$

such that $z(t) = c(t) * g(t)$, where $z(t)$ is also periodic and

$$z(t) = \sum_{k=-\infty}^{\infty} z_k e^{jk\omega_o t},$$

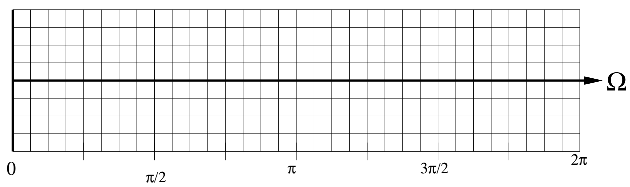
[6 pts] e. Find z_k in terms of $c_k =$ ________________________
(Show your calculation).

Problem 3. DTFT (19 points)

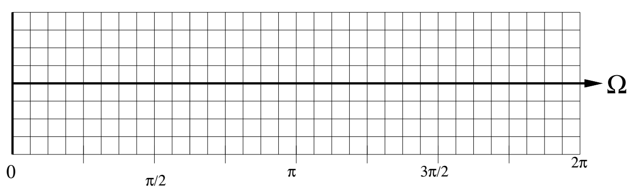
[2 pts] a. Given a discrete time signal $x[n] = \cos(\frac{\pi}{4}n) + \frac{1}{2}\cos(\frac{3\pi}{4}n)$,

find the DTFT $X(e^{j\Omega})$ for $0 \leq \Omega < 2\pi$ =________________

[4 pts] b. Sketch $X(e^{j\Omega})$:



[4 pts] c. A causal LTI system with input $x[n]$ has output $y[n]$. Let $y[n]$ have DTFT $Y(e^{j\Omega})$. Then $Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$. Find and sketch $H(e^{j\Omega})$ such that $y[n] = \cos(\frac{\pi}{4}n)$:



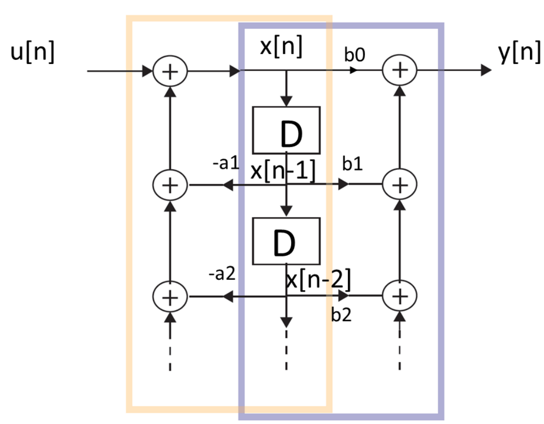
[4 pts] d. Find $h[n]$ for the $H(e^{j\Omega})$ above.

$h[n]$ =_________

[5 pts] e. Given the difference equation for the LTI causal system with input $u[n]$, and output $y[n]$:

$$y[n] = u[n-2] + \frac{3\sqrt{3}}{4}y[n-1] + \frac{9}{16}y[n-2]$$

For the minimal block diagram below, specify coefficients:

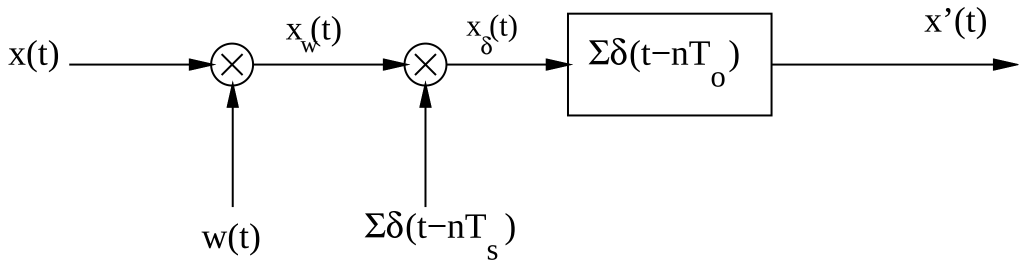


$b_o =$ ___ $b_1 =$ ___ $b_2 =$ ___

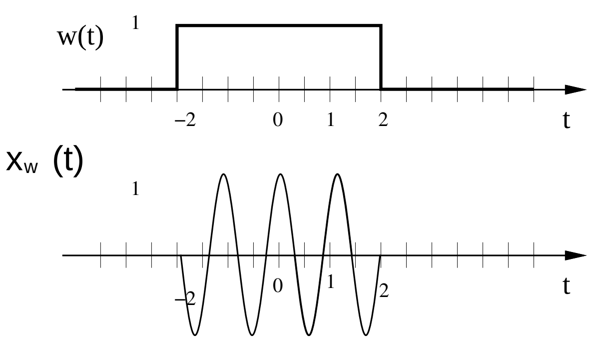
$a_1 =$ ___ $a_2 =$ ___

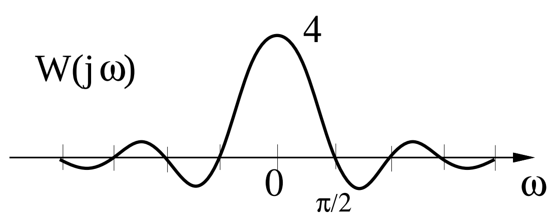
Problem 4. Sampling and Discrete Fourier Transform (24 pts)

Consider the system below, where $x(t) = \cos(\frac{3\pi}{2}t)$. Let $T_s = 0.5$ sec, $T_o = 8$ sec, $w(t) = \Pi(t/4)$. Sketches should label peak magnitudes, and frequency of zero crossing(s) should match given scale. Sketch any sinc functions to 4 zero crossings on each side.
(All time signals are real and even, hence all spectra are also real and even.)
Note $\Pi(t) = u(t+0.5) - u(t-0.5)$.
Note that the window has spectrum $W(j\omega) = \frac{2\sin 2\omega}{\omega}$.



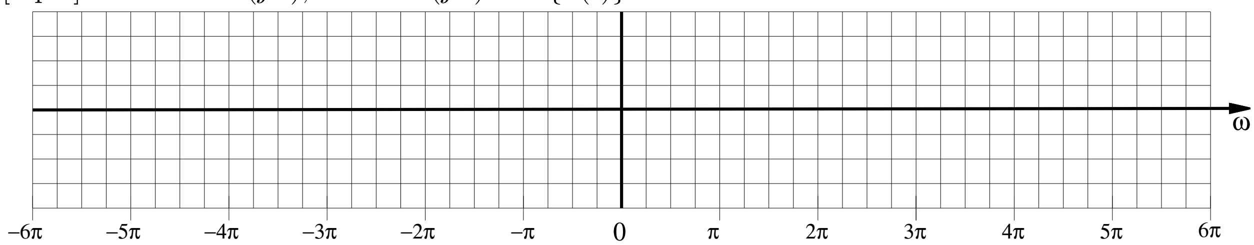
The window function $w(t)$, windowed cosine $x_w(t)$ and $W(j\omega)$ are shown for convenience here:



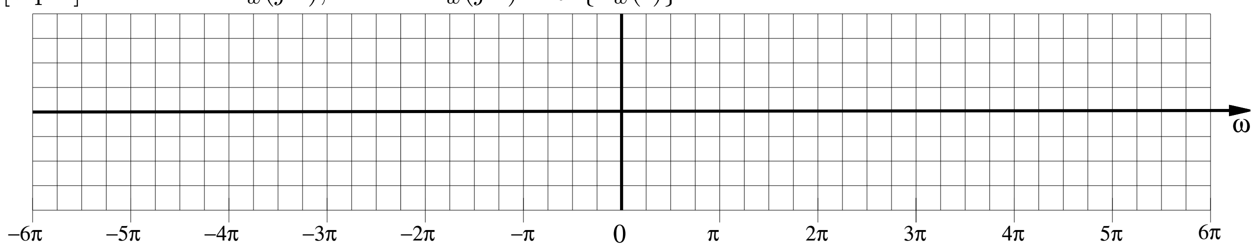


Problem 4, cont.

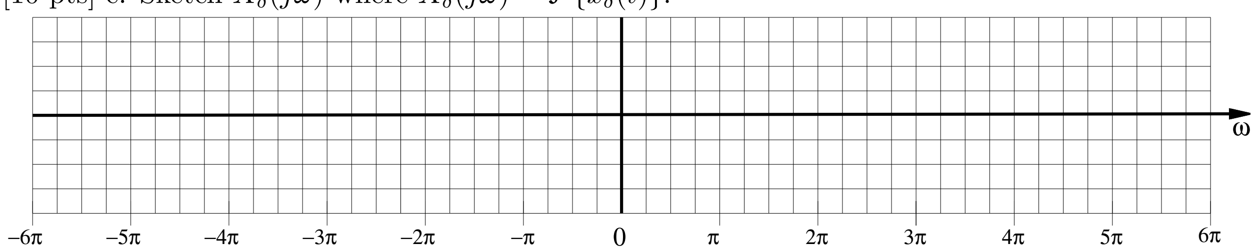
[2 pts] a. Sketch $X(j\omega)$, where $X(j\omega) = \mathcal{F}\{x(t)\}$:



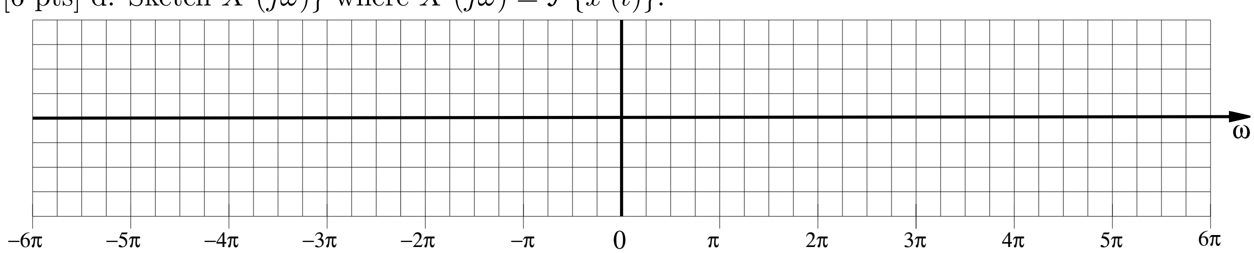
[6 pts] b. Sketch $X_w(j\omega)$, where $X_w(j\omega) = \mathcal{F}\{x_w(t)\}$:



[10 pts] c. Sketch $X_\delta(j\omega)$ where $X_\delta(j\omega) = \mathcal{F}\{x_\delta(t)\}$:

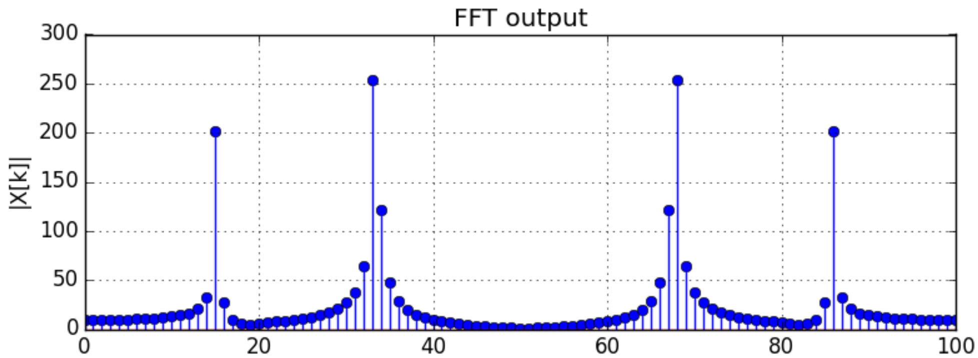


[6 pts] d. Sketch $X'(j\omega)\}$ where $X'(j\omega) = \mathcal{F}\{x'(t)\}$:

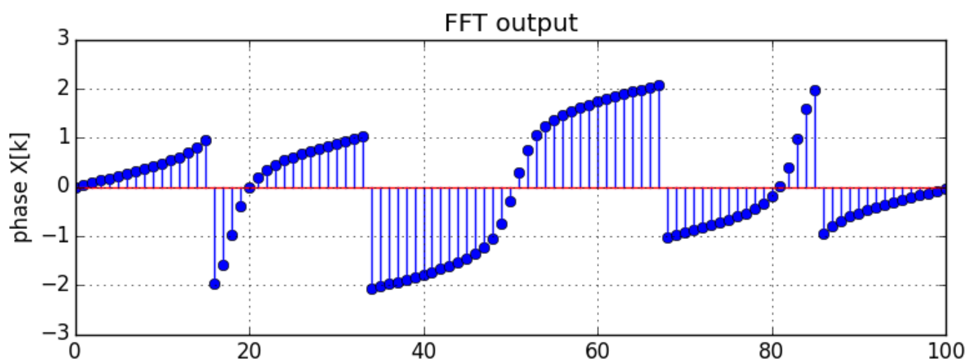


Problem 5. (10 pts)

A real bandlimited signal $x(t)$ is sampled with $N = 100$ for 10 seconds, using a rectangular window of width 10 seconds. The DFT of $x[n]$ is calculated using `X = np.fft.fft(x)`. The magnitude and phase of the DFT is shown below.



$|X[k]|$



$\angle X[k]$

[2 pt] a. What is the spacing of frequency samples $k =$ ___ (rad s^{-1})

Assume $x(t) = a_1 \cos(\omega_1 t + \phi_1) + a_2 \cos(\omega_2 t + \phi_2)$, and that $\omega_1 < \omega_2$.

[4 pt] b. From the DFT plot, estimate ω_1 and ω_2: $\omega_1 =$ ___ $\omega_2 =$ ___
(Note: should be to within 0.5 of spacing of frequency samples.)

[4 pt] c. From the DFT plot, approximately estimate a_1 and a_2: $a_1 =$ ___ $a_2 =$ ___
(Note- you need to consider effect of the rectangular window.)