MIDTERM 2, Fall 2016

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period.

GOOD LUCK!

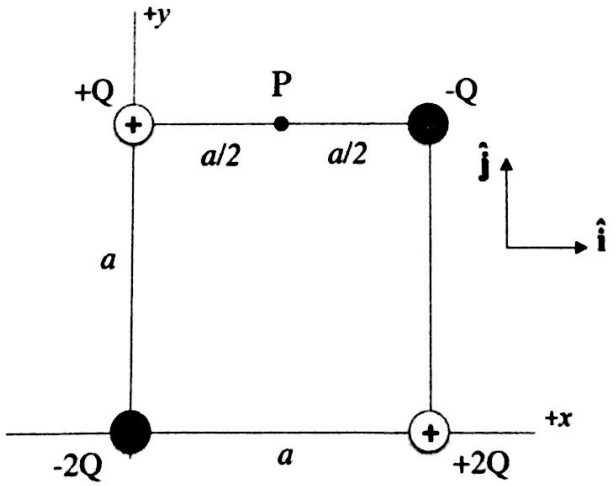
PROBLEM 1 (Points 20)

Four point-like objects of charge -2Q, +Q, -Q and +2Q respectively are placed at the corners of a square of side a, as shown in the figure below.

a) (7pts) Determine the electric field at the point P, which has coordinates (x_P, y_P) = ($a/2, a$). Express your answer in terms of $k=1/4\pi\varepsilon_0$, Q, a, $\hat{\imath}$ and $\hat{\jmath}$ as needed.

b) (7pts) Choose infinity to be the zero point for the electric potential ($V(\infty) = 0$). Determine the electric potential at the point P, which has coordinates (x_P, y_P) = ($a/2, a$). Express your answer in terms of $k=1/4\pi\varepsilon_0$, Q, a, $\hat{\imath}$ and $\hat{\jmath}$ as needed.

c) (6pts) Determine the direction and magnitude of the electric dipole moment of this charge configuration. Express your answer in terms of $k=1/4\pi\varepsilon_0$, Q, a, $\hat{\imath}$ and $\hat{\jmath}$ as needed.



PROBLEM 2 (Points 20)

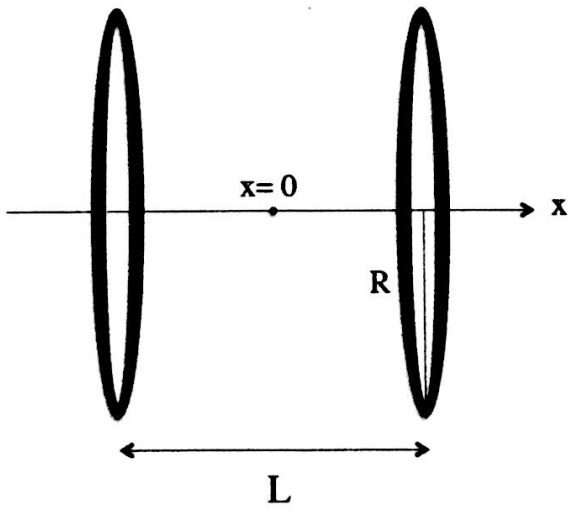
A block in the shape of a rectangular solid has a cross-sectional area A, a length of L and a resistance R. The material of which the block is made has in total N conduction electrons. A potential difference of V_0 is maintained between its ends.

a) (7pts) Find the current in the block.
b) (7pts) Assuming that the current density is uniform, what is its value?
c) (6pts) Calculate the drift velocity of the conduction electrons and the electric field in the block.

PROBLEM 3 (Points 20)

Two parallel circular rings of radius R have their centers on the *x* axis separated by a distance L as shown in the figure below.
The ring on the left carries a uniformly distributed charge +Q, while the one on the right a uniformly distributed charge -Q.

a) (8pts) Take the x-axis to have x = 0 at the midpoint between the rings. Point P is an arbitrary point on the x-axis with position (x, 0, 0). Determine the potential at P.
b) (8pts) Starting with the potential from a) determine the *x, y, z* component of the electric field at the same point.
c) (4pts) Determine the acceleration (magnitude and direction) of a charge q placed at point x = L/2. Assume charge has mass of 2q.

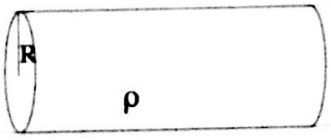


PROBLEM 4 (Points 20)

Consider a charged infinite cylinder of radius R. The charge density is non uniform and given by $\rho(r) = br$, with $r < R$, where r is the distance from the central axis and b is a constant greater than zero.

a) (10pts) Find an expression for the direction and magnitude of the electric field inside and outside the cylinder and discuss any symmetry argument that might be relevant. Show in a figure the direction of the field. Clearly present your reasoning, figures and any accompanying calculations.

b) (10pts) A point-like object with charge +q and mass m is released from rest at a point at distance 2R from the central axis of the cylinder. Find the speed of the object when it reaches a distance 3R from the central axis of the cylinder.



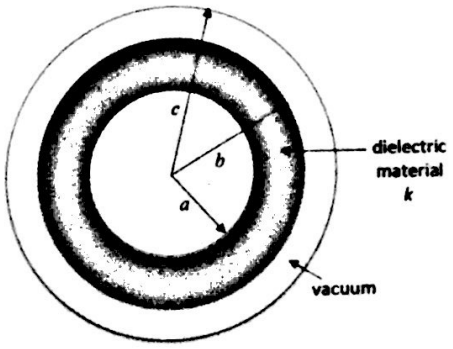
PROBLEM 5 (Points 20)

A spherical capacitor consists of two thin conducting spherical shells and a spherical shell of dielectric material partially filling the space between the conducting shells. The inner conducting shell has radius ***a*** and the outer conducting shell has radius ***c***. The space between is partially filled (from ***a*** out to ***b***) with a material of dielectric constant K. The space between ***b*** and ***c*** is empty (vacuum).

a) (7pt) Determine the capacitance

b) (6pts) When the conducting surfaces of the capacitor are connected to a battery with a potential difference V, determine an expression for the stored energy U.

c) (7pts) While the battery is still connected, the entire region between b and c is now filled with the material of dielectric constant K. By what amount has the magnitude of the free charge changed on either conduction shell $\Delta Q = Q_F - Q_0$?



Physics 7B, Fall 2016: Midterm 2 Formula Sheet

Electrostatics

$$\vec{F} = Q\vec{E}$$

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\epsilon = K\epsilon_0$$

$$C = KC_0$$

$$R = \rho \frac{l}{A}$$

$$P = IV$$

$$I = \int \vec{j} \cdot \vec{dA}$$

Mathematics

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n!\, 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\int (1+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (k^2+x^2)^{-\frac{1}{2}} dx = \ln(x + \sqrt{k^2+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (k^2+x^2)^{-1} dx = \frac{1}{k} \arctan\left(\frac{x}{k}\right)$$

$$\int (1+x^2)^{-\frac{3}{2}} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int (k^2+x^2)^{-\frac{3}{2}} dx = \frac{x}{k^2\sqrt{k^2+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2)$$

$$\int \frac{x}{\sqrt{1+x^2}} dx = \sqrt{1+x^2}$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$