University of California, Berkeley
Department of Mechanical Engineering
ME 104, Fall 2016

Midterm Exam 2 (9 November 2016)

1. Let $\mathbf{F}$ be the resultant force acting on a particle B of mass m and let $\mathbf{G}$ be the linear momentum of B. Denote the linear impulse of $\mathbf{F}$ over a time interval $t_1 \leq t \leq t_2$ by

$$\boldsymbol{\mathcal{I}} = \int_{t_1}^{t_2} \mathbf{F}\, dt, \tag{1}$$

(a) Prove that

$$\mathbf{G}(t_2) = \mathbf{G}(t_1) + \boldsymbol{\mathcal{I}}. \tag{2}$$

(b) A particle of mass $m = 0.2$ kg is traveling on a frictionless horizontal surface with a constant velocity

$$\mathbf{v}_0 = 1.25\, \mathbf{i}\, \mathrm{m/s}. \tag{3}$$

It is desired to change its velocity to 3.75 m/s in a direction that makes an angle of 60° counterclockwise with the unit vector $\mathbf{i}$. Suppose that we are required to achieve this by applying a force of <u>constant</u> magnitude F in an unspecified direction

$$\mathbf{e} = \cos\phi\, \mathbf{i} + \sin\phi\, \mathbf{j}, \quad \phi = \text{const.}, \tag{4}$$

for $\Delta t = 0.4$ s. Solve for F, $\mathbf{e}$, and ϕ.

(c) Indicate your results on a vector diagram.

2. A collar C weighing 9 lbf can slide along a circular rigid rod ABD in a vertical plane with no friction (See Fig.1). The rod is a distance $r = 6$ in from O. The tangents to the rod at A and D intersect at a point E. A linearly elastic spring is anchored at E and is also attached to the collar. The unstretched length of the spring is $l = 6$ in and the spring constant is $k = 24$ lbf/ft. At time $t = 0$, the collar is released from rest at D, where OD makes an angle of 60^o with OB.

(a) Draw free-body diagrams of the collar for the positions D and B, indicating the directions of the unit tangent and unit normal vectors $\mathbf{e}_t$ and $\mathbf{e}_n$ at these points.

(b) Argue that the total mechanical energy of the collar is conserved, i.e.,

$$E = T + V_g + V_e = \text{const.} \tag{5}$$

(c) Solve for the speed of the collar at B.

(d) Solve for the tangential acceleration and the force N exerted on the collar by the rod at B.

(e) Describe in your own words the motion that takes place for $t \geq 0$.

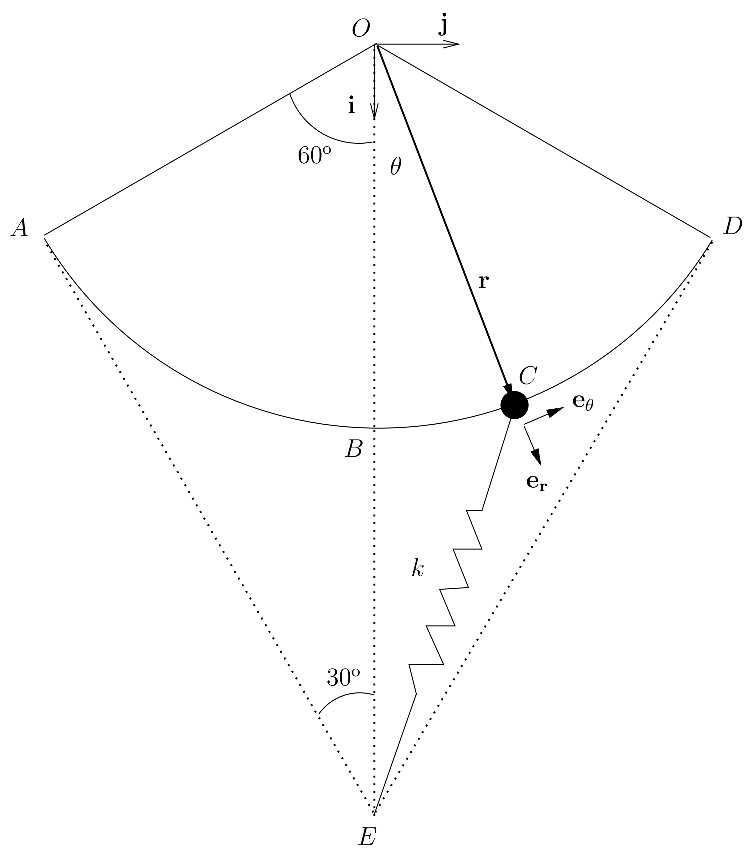

Figure 1

3. The Mars Express satellite has an elliptical orbit about Mars. The minimum and maximum altitudes are 298 km and 10107 km. The mean radius of Mars is $R = 3396$ km. Also, $G = 6.673 \times 10^{-11}$ m^3/(kg $\cdot$ s^2) and the mass of Mars is $M = 0.642 \times 10^{24}$ kg. Let r_p and r_a be the distances from the center of Mars to the lowest and highest points of the orbit, respectively, and let v_p and v_a be the corresponding velocities of the satellite. Recall that the potential energy of the satellite per unit mass is

$$\frac{V}{m} = -\frac{GM}{r}. \tag{6}$$

(a) Prove that

$$v_p^2 = v_a^2 + 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right). \tag{7}$$

(b) Argue that

$$r_p v_p = r_a v_a \tag{8}$$

(c) Calculate v_a and v_p.