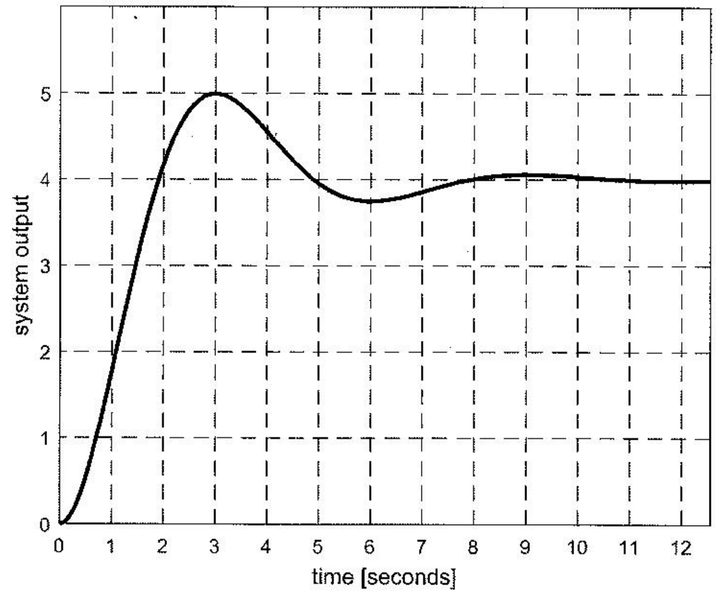
ME 132 Fall 2016 Midterm 1

NAME:

ID # : SOLUTION

# 1	# 2	# 3	# 4	# 5	# 6	#7	TOTAL
9	8	16	12	12	12	12	81

Problem # 1 (3+6 points)



The figure above shows the step response of a second order system with no zeros.

(a) Find the system's percent overshoot, peak time, and DC gain.

(b) Write a procedure for computing the system poles from this information.

a) $\%OS = \dfrac{5-4}{4} \cdot 100 = 25\%$

$T_p = 3$ seconds

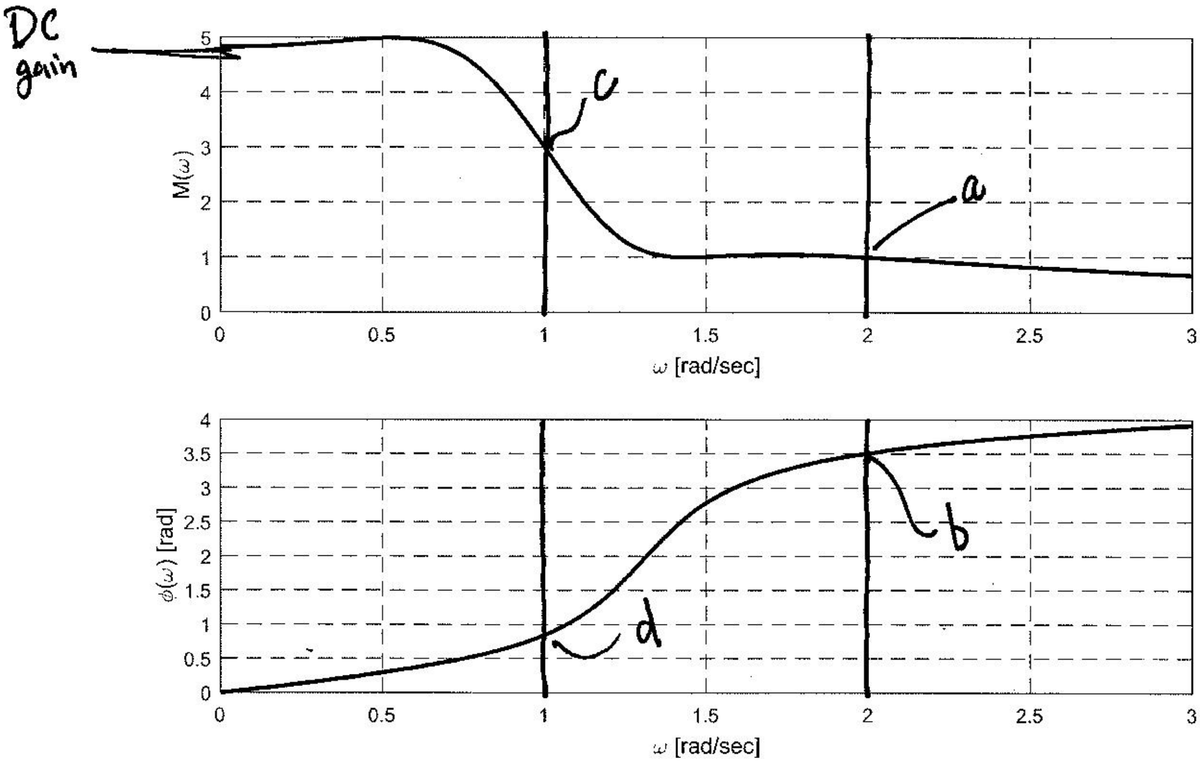
DC gain $= 4$

b) i) $\zeta = -\dfrac{\ln(0.25)}{\sqrt{\pi^2 + \ln^2 0.25}}$

ii) $\omega_n = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}}$

iii) $\lambda_{1,2} = -\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1}$

Problem # 2 (2+3+3 points)



Answer the following questions for this frequency response plot. Notice that all scales are linear. You should define the symbols you need in the figure.

(a) What is the DC gain? (give an approximate numerical answer)

(b) What is the output for an input signal of $4\sin(2t)$?

(c) What input signal produces an output of $12\sin(t)$?

a) DC gain $\cong 4.9$

b) $y(t) = 4a\sin(2t-b)$ with $a \cong 1$, $b \cong 3.5$ rad.

($4a\sin(2t+b)$ is also acceptable)

c) $u(t) = \frac{12}{c}\sin(t+d)$ with $c \cong 3$, $d \cong 0.8$ rad.

($\frac{12}{c}\sin(t-d)$ is also acceptable)

Problem # 3 (2+2+6+6 points)

The next page shows a set of pole plots marked (1) through (4), and a set of step response plots marked (a) through (d). Each of the plots represents 5 different systems. So there is a total of $4 \times 5 = 20$ systems and 20 step responses. Your job is to match them up.

(a) Which of the pole plots represent second order systems? Which represent first order systems?

(b) Which of the step response plots represent second order systems? Which represent first order systems?

(c) For each of the 2nd order pole plots, what quantity do all of the pole-pairs in the plot have in common. Express this as a function of ξ and ω_n.

(d) Fill in the following table.

pole plot	corresponding step response plot (a-d)	common performance metrics ($\%OS$, T_s, T_p). (leave blank for 1st order systems)
(1)	c	%OS
(2)	d	—
(3)	a	T_s
(4)	b	T_p

a) 1, 3, 4 ... 2nd order
2 ... 1st order

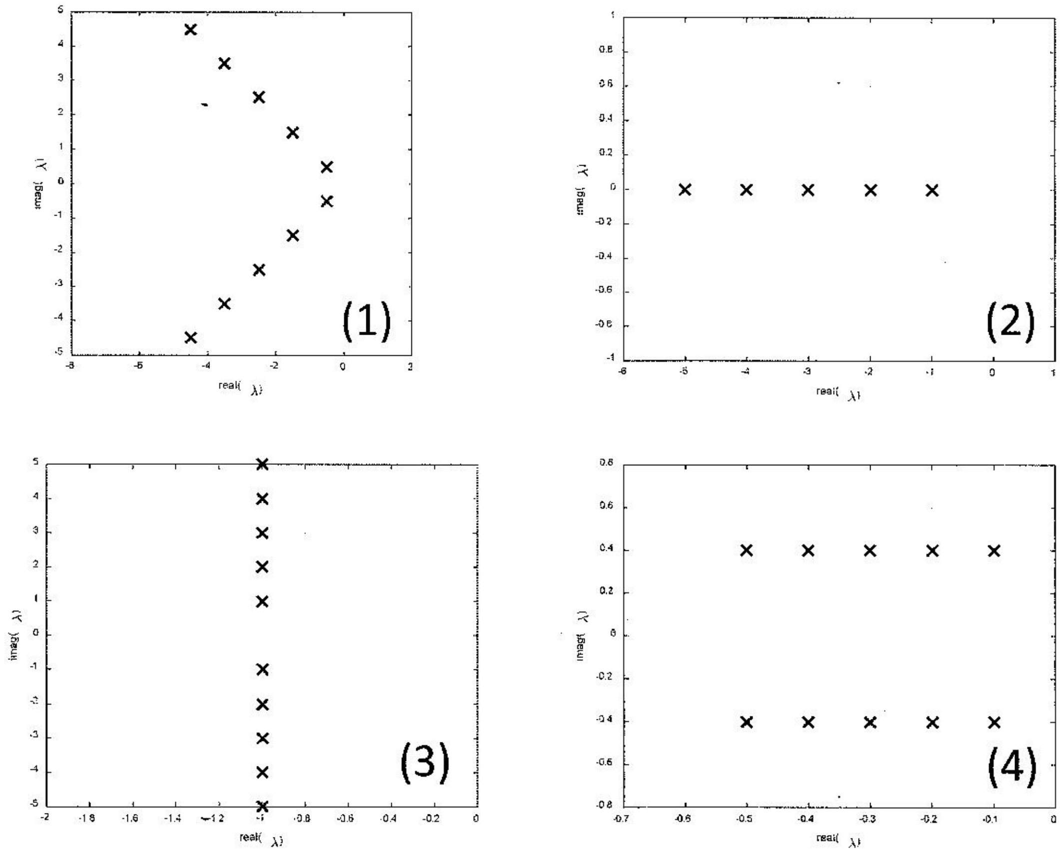
b) a, b, c ... 2nd order
d ... 1st order

c) 1 ... ξ or %OS

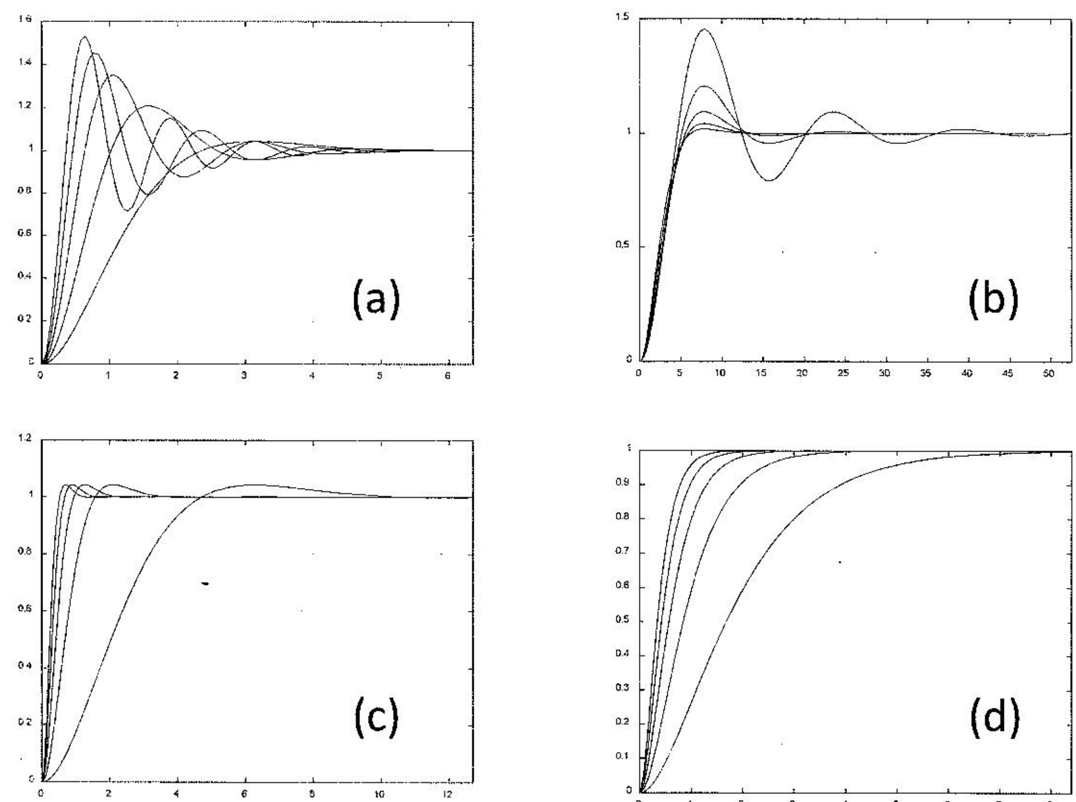
3 ... $\xi \omega_n$ or T_s

4 ... $\omega_n \sqrt{1-\xi^2}$ or T_p

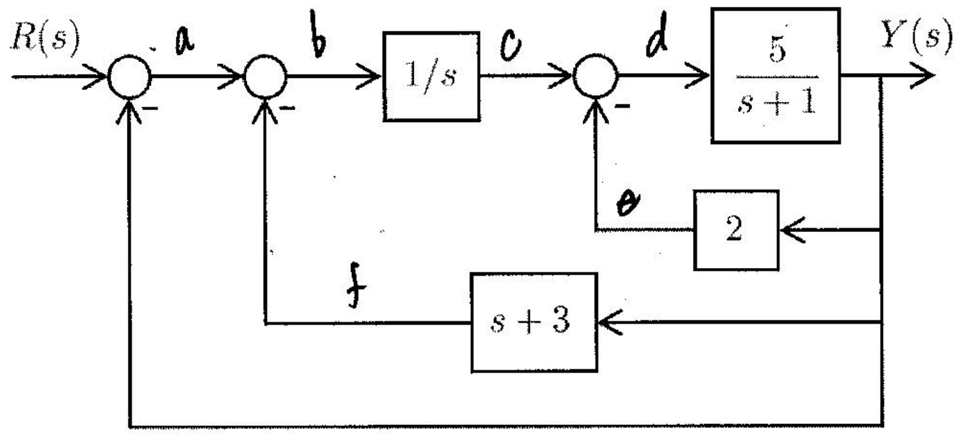
Pole plots:



Step responses:



Problem # 4 (6+4+2 points)



For the block diagram shown above,

(a) Find the transfer function from R to Y.

(b) What type of second order system is this?

(c) Find the DC gain.

a) $$Y(s) = \frac{5}{s+1}(c-e) = \frac{5}{s+1}\left(\frac{1}{s}b - 2Y\right)$$

$$= \frac{5}{s(s+1)}(a-f) - \frac{10}{s+1}Y$$

$$= \frac{5}{s(s+1)}(R-Y) - \frac{5}{s(s+1)}(s+3)Y - \frac{10}{s+1}Y$$

$$\therefore\ s(s+1)Y = 5R - 5Y - 5(s+3)Y - 10sY$$

$$\therefore\ (s^2 + s + 5s + 10s + 5 + 15)Y = 5R.$$

$$\therefore\ \frac{Y}{R} = \frac{5}{s^2+16s+20}$$

b) Denominator : $s^2 + 16s + 20$

$$\omega_n = \sqrt{20} = 2\sqrt{5}$$

$$2\zeta\omega_n = 16$$

$$\therefore \zeta = \frac{8}{2\sqrt{5}} = \frac{4}{\sqrt{5}} > 1 \implies \text{2nd order overdamped}$$

c) $$G(0) = \frac{5}{0^2 + 16(0) + 20} = \frac{1}{4}$$

Problem # 5 (6+3+3 points)

For the system,

$$\dot{y} = -y + 3u$$ ... first order system with $a=-1$, $b=3$

(a) Find an analytical expression for the response $y(t)$ to an input $u(t) = e^{-t}$ with initial condition $y(0) = 1$.

(b) Identify the transient response and the steady state response.

(c) Identify the unforced response and the forced response.

a) $$y(t) = e^{-t}(1) + \int_0^t e^{-(t-\tau)} \cdot 3u(\tau)\, d\tau$$

$$= e^{-t} + 3\int_0^t e^{-t+\tau} e^{-\tau}\, d\tau$$

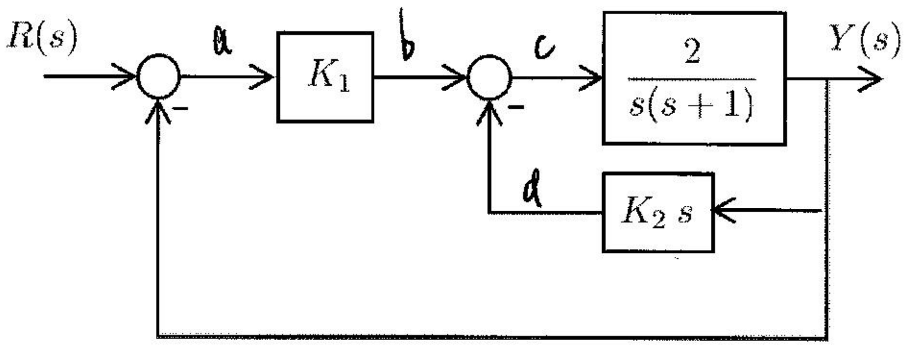
$$= e^{-t} + 3e^{-t}\int_0^t 1\, d\tau$$

$$= e^{-t} + 3te^{-t} = e^{-t}(1+3t)$$

b) $$y(t) = \underbrace{e^{-t}(1+3t)}_{\text{transient}} + \underbrace{0}_{\text{steady state}}$$

c) $$y(t) = \underbrace{e^{-t}}_{\text{unforced}} + \underbrace{3te^{-t}}_{\text{forced.}}$$

Problem # 6 (4+4+4 points)



For the block diagram shown above,

(a) Find the closed loop transfer function from R to Y.

(b) For what ranges of K_1 and K_2 is the system stable?

(c) Compute K_1 and K_2 that produce a closed loop response with $\omega_n = 2$ and $\xi = 1$.

a) $$Y = \frac{2}{s(s+1)}\, c = \frac{2}{s(s+1)}\,(b-d) = \frac{2}{s(s+1)}\,(K_1 a \pm K_2 s Y)$$

$$= \frac{2K_1}{s(s+1)}(R-Y) + \frac{2K_2 s}{s(s+1)}\, Y$$

$$\therefore\ s(s+1)Y = 2K_1 R - 2K_1 Y + 2K_2 s Y$$

$$\therefore\ (s^2 + s + 2K_2 s + 2K_1)Y = 2K_1 R$$

$$\therefore\ \frac{Y}{R} = \frac{2K_1}{s^2 + (1+2K_2)s + 2K_1}$$

b) $$\begin{cases} 1 + 2K_2 > 0 \\ 2K_1 > 0 \end{cases} \iff \quad K_2 > -\frac{1}{2},\quad K_1 > 0$$

c) $\begin{cases} \omega_n = \sqrt{2k_1} = 2 \\ 2\zeta\omega_n = 1 + 2k_2 = 2(1)(2) = 4 \end{cases}$

$\Uparrow$

$$\begin{cases} K_1 = 4/2 = 2 \\ K_2 = +3/2 \end{cases}$$

Problem # 7 (3+6+3 points)

1. An n'th order transfer function has this many state-space realizations:

   (a) 1

   (b) 2

   (c) n

   (d) ∞

2. The first column of the following table provides differential equations for systems with input u and output y. Complete the table with checkmarks in the appropriate cells.

System	is strictly causal	is linear	is time-invariant
$\ddot{y} + 3\dot{y} - 4y = \dddot{u} + 8u$		✓	✓
$\ddot{y} + 3\dot{y} - 4ty = \dot{u} + 8u$	✓	✓	
$\ddot{y} + 3\sqrt{\dot{y}} - 4y = \ddot{u} + 8u$			✓

3. Match the concepts on the left with their mathematical notation on the right (write the letter in the box).

C	sensitivity	A. $\lvert G(0)\rvert$
E	decibels	B. $\int_{-\infty}^{\infty} g(t)h(t-\tau)d\tau$
B	convolution	C. $\frac{dA}{da}\frac{a_0}{A_0}$
A	steady state gain	D. $M(\omega)U\sin(\omega t - \phi(\omega))$
D	frequency response	E. $20\ \log_{10} X$
F	Laplace transform	F. $\int_0^{\infty} e^{-st}f(t)dt$