ECON 101A - MIDTERM 2

Fall 2016. Instructor: Cecile Gaubert

– Do not turn pages unless intructed to. –

Last Name (Please print): _

First Name: _

SID: _

Section you attend: _
(The marked exam will be returned to you in the section that you indicate.)

Instructions. Please read carefully.

- This is a closed-book examination. Put away your books, handouts, notes, calculators, phones and all electronic devices.
- Write your answers in the space provided between questions.
- Write your answers clearly.
- Show your work
- The exam consists of (12) sheets with questions and space for your answers.
  - Pages 1 to 11 have questions and space for your answers
  - Page 12 is an extra blank page if needed
- The last page (double sided) gives you an overview of the whole exam.
- You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 11.00 sharp.
- **Good luck!**

Econ 101A – Midterm 2

Problem 1. Uncertainty(points) Sally has utility function over her income $U(I) = \sqrt{I}$. Her daily income is M. Sally drives to work every day and she spends a lot of money in parking meters. Many days the thought of cheating and not paying for parking crosses her mind. If she cheats and get caught, she has to pay a ticket C. Sally knows that there is a probability p of being caught in a given day if she cheats.

1. What are Sally's preferences towards risk? Is she risk averse, risk loving, or indifferent ("risk neutral")? Explain your answer.
   Answer: Sally is risk-averse—her utility function is concave $U''(I) = -\frac{1}{4}I^{-3/2} < 0$. Both of our measures of risk aversion (relative and absolute) will be negative as a result of this.

2. Write down Sally's expected utility if she cheats.
   Answer: $\mathbf{E}[U] = (1-p)\sqrt{M} + p\sqrt{M-C}$

3. What is the maximum amount of she will be willing to pay for one day parking, as a function of M, p and C
   Answer: We want to find the (maximum) risk premium that Sally is willing to pay (think of the municipality as an insurance company). That is find F^* such that $\sqrt{M-F^*} = (1-p)\sqrt{M} + p\sqrt{M-C}$. This yields

$$F^* = M - \left[(1-p)\sqrt{M} + p\sqrt{M-C}\right]^2$$

4. Paul also faces the same dilemma every single day. His utility function $U(I) = I$. What are Paul's preferences towards risk? Is he risk averse, risk loving, or indifferent ("risk neutral")?
   Answer: Paul is risk-neutral by the same token $U''(I) = 0$

5. Assume that the cost of the ticket is $C = \$36$, the probability of being caught is $p = 1/4$ and both Paul's and Sally's daily income are $M = 100$. The price of one day of parking is \$9.25. Will Paul cheat or pay the parking meter? Will Sally cheat or pay the parking meter?
   Answer: Paul will cheat. His expected utility under the fine is \$91 and his expected utility under the fee is \$90.75. For Sally, we can compare the fee to her maximum fee of

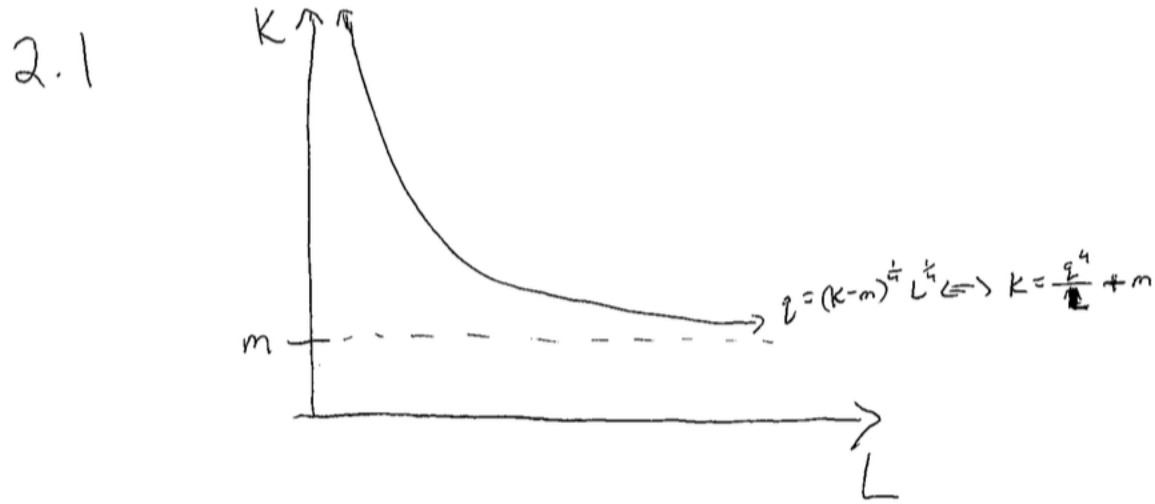
$$\begin{aligned} F^* &= 100 - \left[0.75\sqrt{100} + 0.25\sqrt{64}\right]^2 \\ &= 9.75 \end{aligned}$$

   Because the fee of \$9.25 is smaller than the greatest possible fee of \$9.75, she'll pay the fee rather than risk a ticket. Alternatively, we could compare her expected utility when cheating and when paying the parking fee - you will find that her expected utility in the former case is $0.75\sqrt{100} + 0.25\sqrt{100-36} = 9.5 = \sqrt{90.25}$ and her utility in the later is $\sqrt{100-9.25} = \sqrt{90.75}$. Since her utiltiy is higher when paying the fee, Sally will not cheat.

6. Is this statement true or false? (Explain your answer.) "If Sally has to decide between two different lotteries, she will always prefer a lottery with less risk".
   Answer: False. If Sally was told to choose between a 50-50 lottery between \$1 million and \$10 million and a 100-0 lottery between \$1 and \$1, she'd choose the former.

Problem 2. Cost minimization and monopoly pricing (points) MachineCo produces machines according to the following production function: $q = (K - m)^{1/4} L^{1/4}$, where K is capital, L is labor and m is a parameter. The unit cost of capital is r and the wage is w.

1. Draw a picture of the isoquants. How can one interpret the $-m$ term in the production function ?
   Answer: Please see the image below. The isoquants are downward sloping and convex, and there should be an asymptote at m along the capital axis. The parameter m can be seen as a fixed amount of capital required for production.



2. Derive the MachineCo's demand for inputs—capital and labor, respectively—as a function of its choice of output (q).
   Answer: We want to solve

$$\min_{L,K} wL + rK \quad s.t.\ (K-m)^{1/4} L^{1/4} \geq q$$

Note that our production function is monotonic, so our budget constraint holds with equality. This yields a Lagrangian

$$\min_{L,K} \mathcal{L} = wL + rK - \lambda \left[(K-m)^{1/4} L^{1/4} - q \right]$$

Solving first order conditions of the Lagrangian gives

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = -(K-m)^{1/4} L^{1/4} + q$$
$$\frac{\partial \mathcal{L}}{\partial L} = 0 = w - \left[\frac{1}{4} (K-m)^{1/4} L^{-3/4} \right]$$
$$\frac{\partial \mathcal{L}}{\partial K} = 0 = r - \left[\frac{1}{4} (K-m)^{-3/4} L^{1/4} \right]$$

Manipulating the last two first order conditions gives

$$\frac{w}{r} = \frac{K-m}{L}$$

Substituting this back into the third first order condition, one can get

$$q = \left(\frac{w}{r} L \right)^{1/4} L^{1/4}$$

and thus $L^* = \left(\frac{r}{w}\right)^{1/2} q^2$. Correspondingly, $K^* = m + \left(\frac{w}{r}\right)^{1/2} q^2$

3. What is MachineCo's total cost function $c(q, w, r; m)$?
Answer: Plugging K^* and L^* back into the objective function gives us $c(q, w, r; m) = 2(wr)^{1/2} q^2 + rm$

Assume, from now on, that $r = 1$, $w = 1$ and $m = 8$. Consumers demand for machines is $D = 12 - p$.

4. What would be the price of machines sold by MachineCo if it were a monopoly?
Answer: MachineCo now solves

$$\max_q (12 - q) q - 2(wr)^{1/2} q^2 - rm$$

The first order conditions are

$$\begin{aligned} 0 &= 12 - 2q - 4(wr)^{1/2} q \\ q^* &= \frac{6}{1 + 2(wr)^{1/2}} \end{aligned}$$

Plugging this back into the (inverse) demand equation to get the equilibrium price one gets

$$p^* = 6\frac{1 + 4(wr)^{1/2}}{1 + 2(wr)^{1/2}}$$

Plugging in r, w, and m, we get

$$p^* = 10$$

5. MachineCo operates on a perfectly competitive market where firms makes no profits, and it is desperately trying to bribe the government to pass a law that will make the selling of machines a legal monopoly (and MachineCo the only supplier). How much is MachineCo willing to give in bribe, at most, in exchange for the law to be passed?
Answer: MachineCo is essentially willing to pay its profits under monopoly (minus a tiny bit). Its profits under monopoly are:

$$\begin{aligned} \pi &= p^* q^* - 2(wr)^{1/2} q^{*2} - rm \\ &= 36 \left\{ \frac{1 + 4(wr)^{1/2} - 2(wr)^{1/2}}{\left[1 + 2\left(wr^{1/2}\right)\right]^2} \right\} - rm \\ &= 36 \left\{ \frac{1}{\left[1 + 2\left(wr^{1/2}\right)\right]} \right\} - rm \end{aligned}$$

Plugging in our values we get

$$\pi = 12 - 8 = 4$$

Problem 3. Market equilibrium (points) In this question, the graphs you are asked to draw don't have to be very accurate, but they should represent the magnitudes reasonably clearly. In this problem we will consider the fish industry on the island of San Serife. We aggregate all the other goods into one composite y whose price is 1 Peso (the island's currency).

Part A There are 100 consumers on the island, each with the utility function $U(q, y) = y + 10q - 5q^2$, where q is the consumption of fish and y the consumption of the other good (whose price is 1 Peso). Let p denote the price of fish and let consumers all have income M.

1. Find one consumer's demand function for fish (that is, q expressed as function of p). Do not forget to check that the boundary condition $q \geq 0$ is satisfied.

Answer: Suppose consumers have income M. Then, our maximization problem is as follows:

$$\max_{y,q} y + 10q - 5q^2 \ \textbf{ s.t. } \ y + pq \leq M, 0 \leq y, 0 \leq q$$

Assuming that our budget constraint holds with equality,[1] **we can rewrite the above optimization problem as follows:**

$$\max_{q} (M - pq) + 10q - 5q^2 \ \textbf{ s.t. } \ 0 \leq q \leq \frac{M}{p}$$

First order conditions yield

$$0 = -p + 10 - 10q$$
$$q = \frac{10 - p}{10}$$

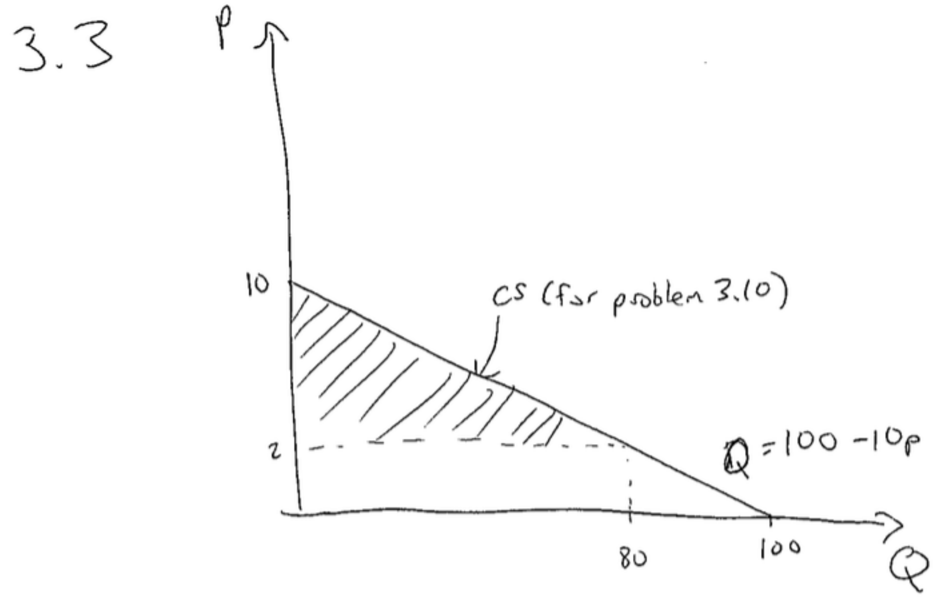
Notice here that quantity of fish does not depend on income. To ensure that the boundary condition is satisfied our individual demand is given by[2]

$$q = \begin{cases} 0 & p > 10 \\ \frac{10-p}{10} & p \leq 10 \end{cases}$$

2. Find the market demand function Q as a function of p.
Answer: Market demand will just be aggregating consumer demands:

$$Q = 100q = 100 - 10p$$

3. Show this in a graph with p on the vertical axis and the market quantity Q on the horizontal axis.
Answer: Shown below. The intercept on the p axis is 10, and the intercept on the Q axis is 100.



[1] It is true that our utility function is not monotonic in q. However, our utility function does represent local nonsatiated preferences, which is sufficient for equality (this was mentioned briefly, although we wouldn't expect you to know this). Additionally, for the values of $p$that solves our utility maximization problem, we notice our utility function is monotonic in q(i.e., $U'_q > 0$ for $0 < q < 1$, and we find those are the values of $q$that solve our utility max problem.

[2] If we were being complete, we should also check the upper boundary, i.e., that $q \leq \frac{M}{p}$. We then find that if $\frac{10-p}{10} > \frac{M}{p} \Leftrightarrow 5 - 2\sqrt{25 - 10M} < p < 5 + 2\sqrt{25 - 10M}$, our actual consumption of q will simply be $\frac{M}{p}$ - otherwise, we'd be spending too much money on fish, or would consume a negative amount of the other good. For the rest of this problem, we will assume that $M \geq \frac{5}{2}$, i.e., M is sufficiently large so that we don't run into this problem.

Part B The fishing industry consists of several firms. Each firm, to produce and sell anything at all, must rent a boat (one boat is always enough for one firm, even if a lot of fish is fished). The cost of renting a boat is 1 Peso. The variable cost of producing output q is: q^2 Peso.

4. Write down expressions for each firm's total cost, average cost and marginal cost (MC), in each case as functions of q.
   Answer: The total cost is the variable cost plus the fixed cost $c = 1 + q^2$. The average cost is $\frac{1+q^2}{q}$ and the marginal cost is $2q$

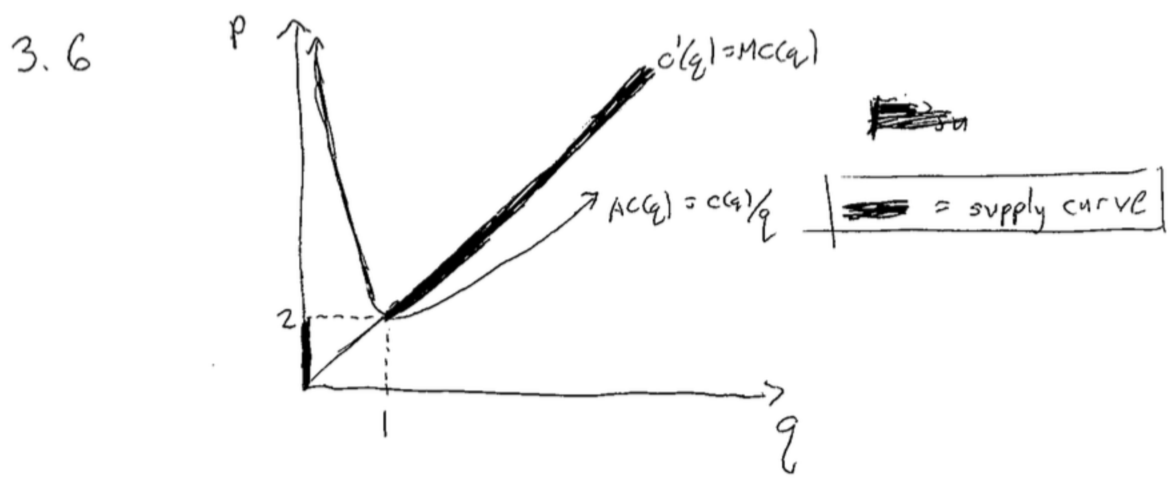
5. Find the equation for the firm's supply curve.
   Answer: Under perfect competition, firms take price as given and set price equal to marginal cost. Therefore, $q^* = \frac{p}{2}$. At that point, average revenue (also price) has to exceed average costs for the firm to at least break even and thus

$$\begin{aligned} \frac{1 + p^2/4}{p/2} &< p \\ 1 + \frac{p^2}{4} &< \frac{p^2}{2} \\ 4 + p^2 &< 2p^2 \\ 2 &< p \end{aligned}$$

   This yields a supply function

$$q = \begin{cases} 0 & p < 2 \\ \frac{p}{2} & p \geq 2 \end{cases}$$

6. Draw rough sketches of the average cost curve, the marginal cost curve and the supply curve.
   Answer: shown below.



Part C In the long run, there is free entry and exit of fishing firms.

7. What is the industry's long run supply curve?
   Answer: The long-run supply curve is given by any q so long as $p = 2$.

8. Suppose the industry is in long run equilibrium. Putting together the market demand curve you found in question 2 and the long-run industry supply curve you just found in question 7, find the long run equilibrium price and quantity.
   Answer: As in the previous part, in long-run equilibrium, the $p = 2$. Aggregate demand yields $Q = 100 - 20 = 80$.

9. Show that 80 firms operate in this equilibrium. What is the profit of each?
   Answer: The easy part of this question is the profit: 0. To find how many firms operate,

we essentially work backward. Each individual firm produces $\frac{2}{2} = 1$ unit, and therefore there must be 80 firms.

10. What is the total of all consumers' surpluses?
    Answer: Note there are no income effects, and so our Marshallian and Hicksian demand curves are the same. We know that demand is zero for prices above 10. From our graphical analysis, we also know that the aggregate demand curve is a line. Therefore, aggregate consumer surplus is $80 \cdot 8/2 = 320$ Pesos.

Part D (can be done independently from part C) Now suppose the government decides to subsidize $s\%$ of the cost of boats for each firm active in the fish industry. The rental cost of boats for firms is now $1 - s$ instead of 1.

11. For each incumbent firm, what is its new cost function, marginal cost function and average cost function, as a function of s?
    Answer: The new total cost is $c = 1 - s + q^2$. The average cost is $\frac{1-s+q^2}{q}$ and the marginal cost is $2q$

12. The governement decides to levy a tax of 0.25 Peso per unit of fish. Suppose that the number of firms is fixed in the short run, at 80 firms as shown in Part C, and that they are all active on the market (Do not worry at this stage about whether firms make positive profits or not.) What is the price paid by the consumers? What is the price received by the firms?
    Answer: We want to equalize supply and demand in the presence of the tax. Ignoring the extensive margin decision of the firm,

$$\begin{aligned} 100 - 10\,(p + 0.25) &= 40p \\ 97.5 &= 50p \\ p &= 1.95 \end{aligned}$$

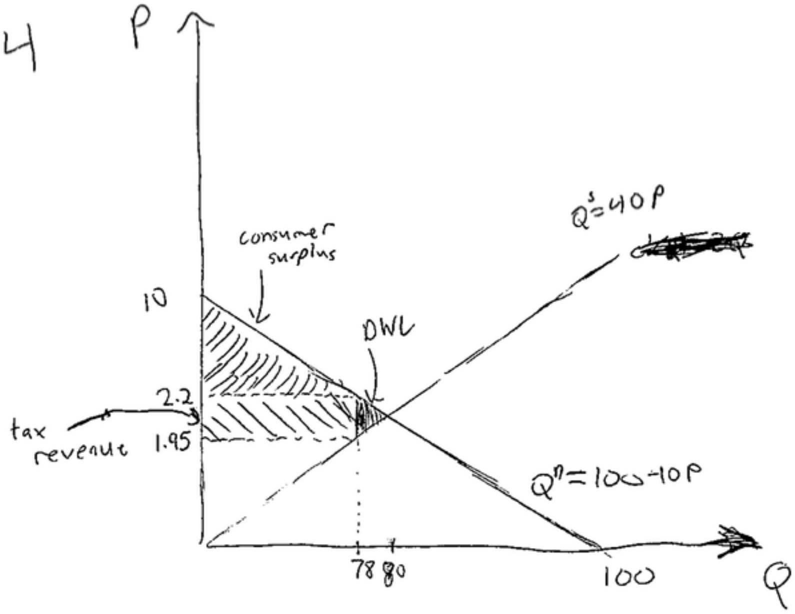
    This is the price the firm sees. The price the consumer sees is 2.20 Pesos/fish. [3]

13. What is the total quantity demanded by consumers? How much does each firm has to supply to satisfy this demand ? Write down how you would find the minimum subsidy of the cost of boats s such that firms break even. (you don't have to find how much it is, just write the equation that you would have to solve to find it).
    Answer: Total quantity can be gotten by sticking 2.20 back into either supply or demand. This yields $Q = 78$. Each firm supplies $q = \frac{78}{80}$. In order for firms to break even, profits need to be zero. We can write this as saying average cost equals price, or $\frac{1+q^2-s}{q} = p$. If we solve this, we find that $s = 0.049$ pesos.

14. Show graphically how to find the short run consumer surplus, how much tax revenue the government collects, and the dead-weight loss from the tax.
    Answer: Shown below.

[3] Here, we solved $Q^D(p+t) = Q^S(p)$,where $p$is the producer price and $p+t$ is the consumer price. This is the same as solving $Q^D(p) = Q^S(p-t)$, where p is the consumer price and $p-t$ is the producer price.

3.14


P
Q
consumer surplus
DWL
10
2.2
1.95
tax revenue
$Q^s = 40P$
$Q^D = 100 - 40P$
78
80
100