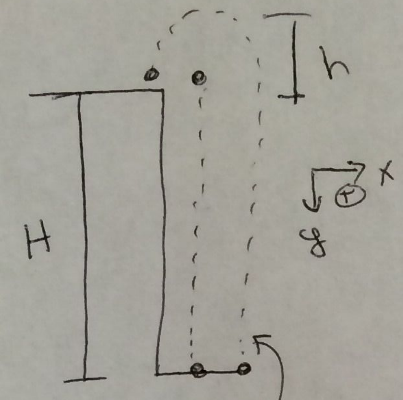
Q1

→ Solve for H & g_0

→ $t_1 = 2.0s$, $t_2 = 2.4sec.$



Note, second
trajectory does not
ave an x-component
ut is drawn here
not coincide with
e first trajectory.

Step 1

→ use $\Delta y = v_{o,y}(t) + \frac{1}{2}at^2$ equation in order to set up system of equations

(1) $H = \frac{1}{2} g_0 t_1^2$

(2) $H = -v_{o,y} t_2 + \frac{1}{2} g_0 t_2^2$

Step 2

→ Solve for $v_{o,y}$ using $v_f^2 = v_0^2 + 2a\Delta x$ equation

→ Set v_f to be at top of second trajectory.

→ $v_f = 0$, $h = 1$

→ $0 = v_{o,y}^2 + 2g_0(-h)$

(3) $v_{o,y} = \sqrt{2g_0 h}$

Step 3

Plug in (3) into (2) and set (1) = (2) as H = H , solve for g_0

$$\frac{1}{2} g_0 t_1^2 = -\sqrt{2g_0 h}\, t_2 + \frac{1}{2} g_0 t_2^2$$

$$\left[\frac{1}{2} g_0 (t_1^2 - t_2^2)\right]^2 = \left(-\sqrt{2g_0 h}\, t_2\right)^2$$

$$\frac{1}{4} g_0^{\cancel{2}} (t_1 - t_2)^2 = 2\cancel{g_0} h t_2^2$$

$$g_0 = \frac{2 h t_2^2}{\frac{1}{4}(t_1 - t_2)^2}$$

→ Plug in all numbers

$$\boxed{g_0 = 14.88 m/s^2}$$

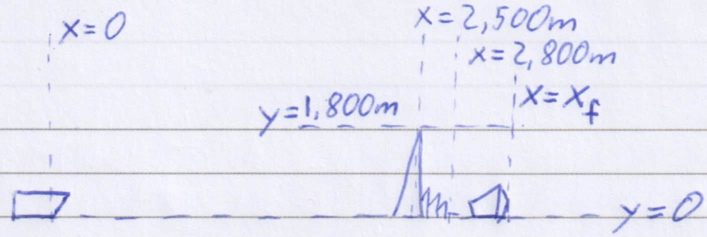
Step 4

→ use (1) and g_0 to solve for H

$$H = \frac{1}{2}(14.88)(2)^2$$

$$\boxed{H = 29.75 m}$$

Physics 7A Midterm I
Section 2 (Yildiz), Fall 2015
Problem 2, Page 1



Trajectory for the projectile:

$$x = v_{0x} t \qquad y = v_{0y} t - \tfrac{1}{2} g t^2$$

We know that the projectile passes these points:

$\langle 2{,}500\,m, 1{,}800\,m \rangle$ and $\langle x_f, 0 \rangle$.

The goal is to find x_f.

We also know that $v_{0x} = (250\,m/s) \cdot \cos\theta$ and $v_{0y} = (250\,m/s) \cdot \sin\theta$.

We know nothing about time, let's eliminate it from the equation.

$$t = \frac{x}{v_{0x}} \Rightarrow y = v_{0y} \cdot \frac{x}{v_{0x}} - \frac{1}{2} g \frac{x^2}{v_{0x}^2} = \tan\theta \cdot x - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}.$$

We need to plug in values from the first point to find θ.

$$1{,}800\,m = \tan\theta \cdot 2{,}500\,m - \frac{1}{2} \frac{9.8\,m/s^2}{(250\,m/s)^2} \cdot \frac{(2{,}500\,m)^2}{\cos^2\theta}$$

$$1{,}800\,m = \tan\theta \cdot 2{,}500\,m - 490\,m \cdot \frac{1}{\cos^2\theta}$$

The answer to this equation is $\theta = 50.13°$ or $\theta = 75.63°$.
I will go over how to solve this equation at the end.
We are instructed by the problem to choose the larger θ.

Now we can use the range formula to find x_f:

$$x_f = \frac{v_0^2}{g} \sin(2\theta) = 3066.5\,m.$$

The shore is located at $x = 2{,}800\,m$. Finding the distance to the shore:

$$\Delta x = x_f - 2{,}800\,m = \underline{266.5\,m}.$$

The ship is safe for 266.5m meters from the shore.

Solving for θ requires ingenuity. Notice that we have both $\sin(\theta)$ and $\cos(\theta)$ in the equation.

$$1,800m = \tan\theta \cdot 2,500m - 490m \cdot \frac{1}{\cos^2\theta}$$

$$\Rightarrow 1,800m \cdot \cos^2\theta = 2,500m \cdot \cos\theta \sin\theta - 490m.$$

Using $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\cos\theta \sin\theta = \frac{\sin 2\theta}{2}$, we find

$$900m + 900m \cdot \cos 2\theta = 1,250m \cdot \sin 2\theta - 490m.$$

$$\Rightarrow \cos 2\theta = 1.389 \cdot \sin 2\theta - 1.544$$

Squaring the equation:

$$\cos^2 2\theta = 1.929 \cdot \sin^2 2\theta - 4.289 \cdot \sin 2\theta + 2.384$$

Using $\cos^2 2\theta = 1 - \sin^2 2\theta$:

$$0 = 2.929 \cdot \sin^2\theta - 4.289 \cdot \sin 2\theta + 1.384$$

$$\Rightarrow 0 = \sin^2 2\theta - 1.464 \cdot \sin 2\theta + 0.473$$

Solving the quadratic, we find

$\sin 2\theta = 0.481$ or $\sin 2\theta = 0.983$, which corresponds to

$\theta = 75.6^\circ$ or $\theta = 50.1^\circ$.

Problem 3 Official Rubric

10 points

Student would have to understand the time it takes for an object to land on the incline is only a function of how high it goes normal to the incline (tilting the the picture such that the incline is now the horizontal). W.r.t the incline projectile B has a smaller height compared to A, so B would hit the ground first.

7 points

Hang time is based on the max height of the projectile but since B lands on an incline. B would land first.

5 points

Assumed same initial velocity however had the understanding that flight time was based on smaller angle w.r.t. to the incline plane.

3 points

Time of flight a function of theta and also assumed the initial velocities were equal

2 points

Time of flight is a function of theta and also assumed the initial velocities were equal.

0 points

Time of flight is a function of theta. (only possible if initial velocities are equal)

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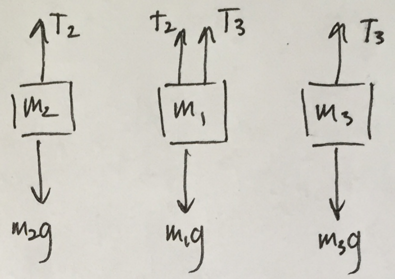
#4 let's draw out the FBDs:



note that there are two separate tensions since there are two unconnected strings.

Now, Newton's 2nd Law gives the eqns of motion:

$$m_1 a = m_1 g - T_2 - T_3 \qquad (1)$$

$$m_2 a = T_2 - m_2 g \qquad (2)$$

$$m_3 a = T_3 - m_3 g \qquad (3)$$

note all the accelerations of the masses are the same because they are one big connected system.

Then solve for a by adding: (1) + (2) + (3) : $(m_1 + m_2 + m_3)\, a = (m_1 - m_2 - m_3)\, g$

$$\boxed{a = \frac{m_1 - m_2 - m_3}{m_1 + m_2 + m_3}\, g}$$

plugging in numbers gives: $\boxed{a = 1.09 \, m/s^2}$

($m_1 = 40$ kg, $m_2 = 20$ kg, $m_3 = 12$ kg, $g = 9.8 \, m/s^2$)

note that $a = \frac{10}{9} \, m/s^2 = 1.11 \, m/s^2$ is also accepted.

Suppose m_1 and m_2 slide together. Then we have:

$$a(m_1 + m_2) = T$$

$$am_3 = m_3 g - T$$

$$a(m_1 + m_2 + m_3) = m_3 g$$

$$a = g\frac{m_3}{m_1 + m_2 + m_3} = \frac{4}{9}g$$

The friction force required to accelerate m_2 must not exceed the maximum force the static friction can provide.

$$F_\mu = \frac{4}{9}gm_2 \leq m_1 g\mu_s$$

$$\frac{4}{9}\frac{m_2}{m_1} = \frac{2}{3} \leq \mu_s$$

The above is false, so m_1 and m_2 move independently from each other.

$$a_1 m_1 = T - \mu m_1 g$$

$$a_1 m_3 = m_3 g - T$$

$$a_1(m_3 + m_1) = g(m_3 - \mu m_1)$$

$$a_1 = g\frac{m_3 - \mu m_1}{m_3 + m_1} = g\frac{8 - 0.5 \times 4}{8 + 4} = \frac{g}{2}$$

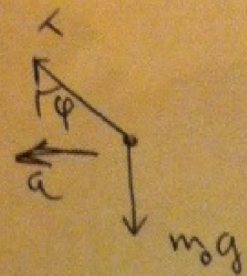
$$a_2 m_2 = \mu m_1 g$$

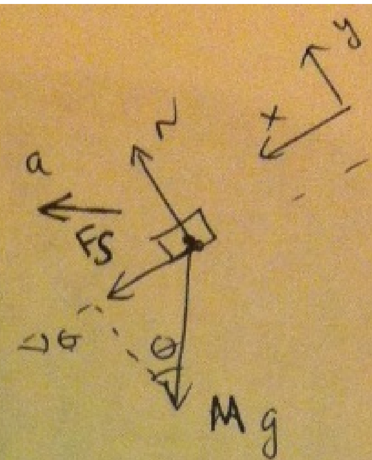
$$a_2 = g\frac{0.5 \times 4}{6} = \frac{g}{3}$$

$$T = (g - a_1)m_3 = \frac{g}{2}m_3 \approx 40 \text{ N}$$

6)

$\varphi = 45°$





$\theta = 30°$

$T\cos\varphi = m_0 g$

$T\sin\varphi = ma$

$\Rightarrow \dfrac{a}{g} = \tan\varphi$

$a = g\tan\varphi = g.$

$\sum F_x = ma_x$

$F_s + Mg\sin\theta = Ma\cos\theta$

$F_s = Mg(\cos\theta - \sin\theta)$

$F_s = 10000\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)$

$= 3660\ (N).$