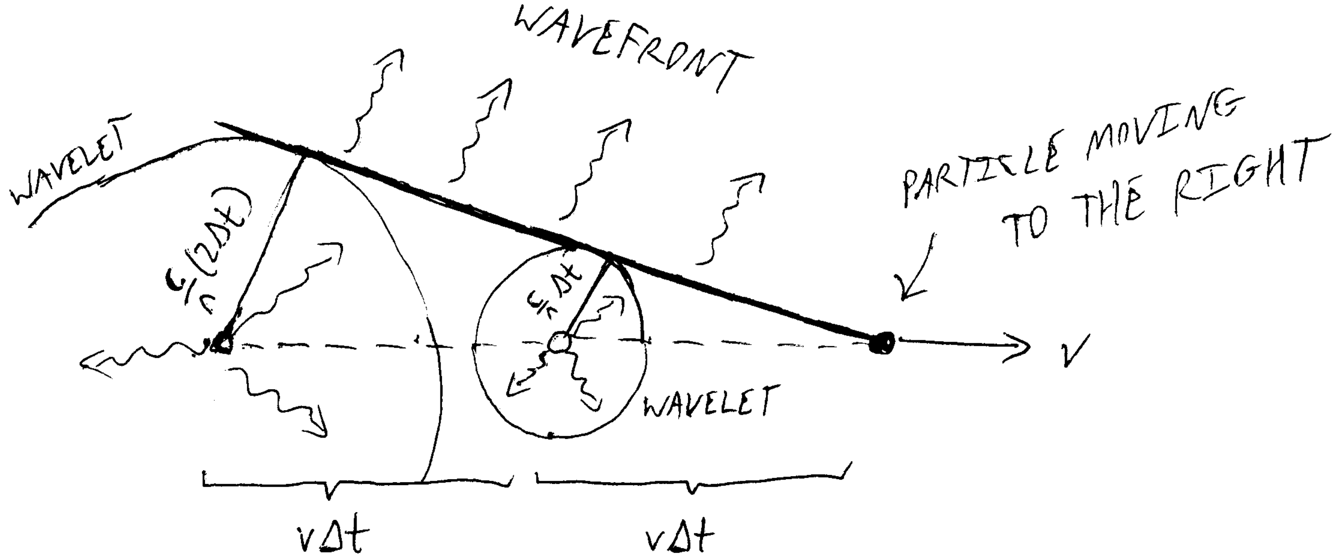
7C FINAL

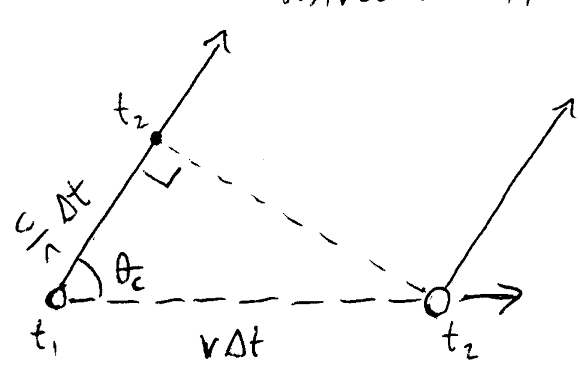
S '04 BATTAGLIA

PROBLEM A1

12pts i) LOOK AT TIME SLICES Δt



WAVELET RAYS



$$\boxed{\cos\theta_c = \frac{\frac{c}{n}\Delta t}{v\Delta t} = \frac{1}{\beta n}}$$

A1 CONT.

5 pts ii) $\beta = \frac{1}{n \cos\theta_c}$

MINIMIZED WHEN $\theta_c = 0$ AND $\cos\theta_c = 1$

$$\boxed{\beta_{min} = \frac{1}{n}}$$

8 pts iii) $\beta = \frac{1}{1.00044}$

$$\gamma = \left[1 - \left(\frac{1}{1.00044}\right)^2\right]^{-1/2} = 33.72$$

$$E = \gamma m_p c^2 = (33.72)(938\ \mathrm{MeV}/c^2)c^2 = \boxed{3.163 \times 10^4\ \mathrm{MeV}}$$

7C FINAL
S '04 BATTAGLIA
PROBLEM A2

6 pts. i) NEED TO PRODUCE A SYSTEM WITH INVARIANT MASS $= M_{Z'}$

i.e., $E_{tot}^2 - p_{tot}^2 c^2 = M_{Z'}^2 c^4$

energy E
momentum $p = \frac{1}{c}\sqrt{E^2 - m_e^2 c^4}$

energy $m_e c^2$
momentum 0

$$E_{tot} = E + m_e c^2$$

$$p_{tot} = \frac{1}{c}\sqrt{E^2 - m_e^2 c^4}$$

$$(E + m_e c^2)^2 - \left(\frac{1}{c}\sqrt{E^2 - m_e^2 c^4}\right)^2 c^2 = M_{Z'}^2 c^4$$

$$\cancel{E^2} + 2E m_e c^2 + m_e^2 c^4 - \cancel{E^2} + m_e^2 c^4 = M_{Z'}^2 c^4$$

$$2E m_e c^2 = (M_{Z'} c^2)^2 - 2(m_e c^2)^2$$

$$E = \frac{(M_{Z'} c^2)^2 - 2(m_e c^2)^2}{2 m_e c^2}$$ NEGLIGIBLE

$$= \frac{(500 \times 10^3\ \text{MeV})^2 - 2(0.511\ \text{MeV})^2}{2(0.511\ \text{MeV})}$$

A2 CONT.

$$\boxed{E = 2.446 \times 10^{11}\ \text{MeV}}$$

6 pts ii)

$e^+ \bullet\!\longrightarrow \quad \longleftarrow\!\bullet\ e^-$

energy E | ~~momentum~~ energy E
momentum $\frac{1}{c}\sqrt{E^2 - m_e^2 c^4}$ | momentum $-\frac{1}{c}\sqrt{E^2 - m_e^2 c^4}$

$$E_{tot} = 2E$$

$$P_{tot} = 0$$

$$(2E)^2 - 0^2 c^2 = M_{Z'}^2 c^4$$

$$2E = M_{Z'} c^2$$

$$E = \frac{1}{2} M_{Z'} c^2$$

$$= \boxed{250\ \text{GeV}}$$

7pts iii) CASE i) MINIMUM INPUT ENERGY WHEN BOTH e^+ AND e^- HAVE 10 GeV/c

$$E_{final} = 2\sqrt{p^2c^2 + m_e^2 c^4} \simeq 2pc \quad \text{(ULTRA-RELATIVISTIC } pc \gg m_e c^2\text{)}$$

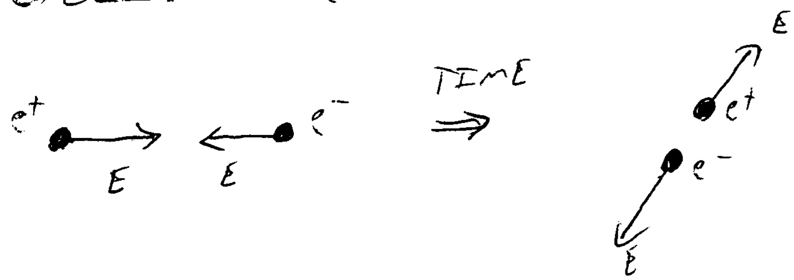
$$E_{initial} = E_{final} = E + m_e c^2 = 20\ \text{GeV}$$

SO $\boxed{E \simeq 20\ \text{GeV}}$

A2 CONT.

CASE ii) MOMENTUM BALANCES $\Rightarrow$ PARTICLES RETAIN ORIGINAL ENERGIES AFTER COLLISION (JUST CHANGE ORIENTATION)

e.g.,



SO $\boxed{E = 10\ \text{GeV}}$

unit volts

6 pts iv) $r = \dfrac{p}{eB} = \dfrac{10\ \text{GeV}/c}{e\ (1T)} = 10^{10}\ \dfrac{V/(3 \times 10^8\ m/s)}{T} = \boxed{33.3\ m}$

electron charge cancels

unit tesla

7C FINAL

5'04 BATTAGLIA

PROBLEM A3

25 pts total

MOVING IN THE −X DIRECTION

$E_\gamma = 3\times10^{-10}\,\text{MeV}$ $\qquad$ E_p

$p_\gamma = \frac{E_\gamma}{c}$ $\qquad$ $p_p \simeq \frac{-E_p}{c}$ (ULTRA-RELATIVISTIC)

NEED TO CREATE A SYSTEM WITH INVARIANT

MASS $= M_p + M_\pi = 938\,\frac{\text{MeV}}{c^2} + 139\,\frac{\text{MeV}}{c^2}$

SOLVE FOR E_p

$$E_{tot} = E_\gamma + E_p$$

$$P_{tot} = \frac{1}{c}(E_\gamma - E_p)$$

$$(E_\gamma + E_p)^2 - \left(\frac{1}{c}(E_\gamma - E_p)\right)^2 c^2 = (m_p + m_\pi)^2 c^4$$

$$\cancel{E_\gamma^2} + 2E_\gamma E_p + \cancel{E_p^2} - \cancel{E_\gamma^2} + 2E_\gamma E_p - \cancel{E_p^2} = (m_p + m_\pi)^2 c^4$$

$$E_p = \frac{(m_p c^2 + m_\pi c^2)^2}{4E_\gamma} = \frac{(938\,\text{MeV} + 139\,\text{MeV})^2}{4(3\times10^{-10}\,\text{MeV})}$$

$$= \boxed{9.666\times10^{14}\,\text{MeV}}$$

B1

$\Delta p \Delta x \sim \hbar$

USE $L \gtrsim \Delta x$, $|p| \gtrsim \Delta p$, $p = m\gamma v = m\gamma\beta c$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \rightarrow |\beta| = \sqrt{1 - \frac{1}{\gamma^2}}$

$|p| = mc\sqrt{\gamma^2 - 1}$

$\rightarrow mc\sqrt{\gamma^2-1} \cdot L \gtrsim \hbar \rightarrow L \gtrsim \frac{\hbar}{mc} \frac{1}{\sqrt{\gamma^2-1}}$

$\rightarrow L \gtrsim \frac{\lambda_c}{2\pi} \frac{1}{\sqrt{\gamma^2-1}}$

FOR $\frac{m\gamma}{m} = \gamma = 1.05$;

$L = \frac{2.43e\text{-}12\ m}{2\pi} \cdot \frac{1}{\sqrt{1.05^2 - 1}}$

$\boxed{L = 1.21e\text{-}12\ m}$

BOHR RADIUS $= \frac{a_0}{Z} \simeq \frac{5e\text{-}11\ m}{Z}$

$\rightarrow Z \gtrsim 40$ HAVE "RELATIVISTIC" INNER SHELL ELECTRONS

B2

CURRENT AROUND LOOP

AREA OF CURRENT LOOP IN X-Y PLANE (FOR FLUX)

(i) $\mu_z = I A = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{q}{2} vr$

(12 pts)

$= \frac{q}{2m}(mvr) = \frac{q}{2m} L_z \rightarrow \frac{q g^L}{2m} L_z$

GYROMAGNETIC RATIO

FOR ELECTRON SPIN:

$q \rightarrow (-e),\ g \simeq 2,\ (m = m_e),\ L_z = S_z = m_s \hbar = \pm \frac{\hbar}{2}$

$\rightarrow \ulcorner \mu_z = \pm \frac{e\hbar}{2m} = \pm \mu_B \lrcorner$

MAGNETIC: $V_B = -\vec{\mu} \cdot \vec{B} = \frac{\mu_B}{\hbar}(g_S \vec{S} + g_L \vec{L}) \cdot \vec{B} = \mu_B (2m_s + m_l) B_z$

COULOMB: $V_E = \frac{k(Ze)(-e)}{r} = \frac{ke^2}{r}$

CENTRIFUGAL: $F_c = \frac{mv^2}{r} = \frac{L^2}{mr^3} \rightarrow V_c = \frac{L^2}{2mr^2} = \frac{l(l+1)\hbar^2}{2mr^2}$

$\rightarrow V_c + V_E + V_B =$

$$\ulcorner V = \frac{l(l+1)\hbar^2}{2mr^2} - \frac{ke^2}{r} + \mu_B B_z (m_l + 2m_s) \lrcorner$$

(OPTIONAL — IS REALLY A K.E. TERM)

(ii) IGNORING SELF-ENERGY OF ORBIT,
(6 pts)

$$V_B = \mu_B B_z \cdot 2 m_s$$

$$\Delta E = V_B(m_s = \tfrac{1}{2}) - V_B(m_s = -\tfrac{1}{2}) = 2\mu_B B_z$$

$$\rightarrow B_z = \frac{\Delta E}{2\mu_B} = \frac{4.5e{-5}\ eV}{2\cdot(5.79e{-5}\ eV\ T^{-1})} = \boxed{0.389\ T}$$

(iii) $B_z \sim \frac{I}{r} \sim \frac{e(+v)}{r}$ ← PROTON MOVING IN RIGHT-HANDED DIRECTION FROM ELECTRON'S POINT OF VIEW (SAME AS ELEC.)
(7 pts)

$$\frac{v}{r} = \frac{mv}{mr} = \frac{mvr}{mr^2} = \frac{L_z}{mr^2}$$

$$\rightarrow B_z \sim L_z \rightarrow \vec{B} \sim \vec{L}$$

$$V_B \sim \vec{S}\cdot\vec{B} \sim \vec{S}\cdot\vec{L}$$

$$\vec{J} = \vec{L} + \vec{S} \rightarrow J^2 = L^2 + 2\vec{L}\cdot\vec{S} + S^2 \Rightarrow \vec{S}\cdot\vec{L} = \frac{1}{2}(J^2 - L^2 - S^2)$$

$$\Rightarrow \vec{S}\cdot\vec{L} = \frac{1}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]$$

FOR HYDROGEN 2P, $\ell = 1$ & $s = 1/2$, SO L^2, S^2 CONST.

$\rightarrow V_B \sim j(j+1) \Rightarrow$ $\boxed{j = 3/2 \text{ HAS HIGHER ENERGY}}$

$\boxed{\text{7C FINAL}\\ \text{S'04 BATTAGLIA}\\ \text{PROBLEM B3}}$

15 pts i) $U(r) = -\frac{ke^2}{r}$ Coulomb potential

3D Schrödinger (time-independent)

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(r,\theta,\phi) + U(r)\,\psi(r,\theta,\phi) = E\,\psi(r,\theta,\phi)$$

with $\mu = \frac{m_e^2}{m_e+m_e} = \frac{1}{2}m_e$ reduced e^+e^- mass

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}(\text{angular derivatives})$$

if you want the full analysis, read the text

for s-wave ($l=0$) solutions, $\psi(r,\theta,\phi) = (\text{const})\,R(r)$, and the angular derivatives disappear, leaving

$$\boxed{-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) - \frac{ke^2}{r}R(r) = E\,R(r)}$$

for $l \neq 0$, there is also a term $\boxed{\frac{l(l+1)\hbar^2}{2\mu r^2}R(r)}$

B3 CONT

5pts ii) this is just hydrogen with $m_e \rightarrow \mu = \frac{1}{2} m_e$

so $\boxed{\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi}} a_0^{-3/2} e^{-r/a_0}}$

with $a_0 = \frac{\hbar^2}{\mu k e^2} = \frac{2\hbar^2}{m_e k e^2}$

double the hydrogen Bohr radius

5pts iii) $\boxed{E_n = -\frac{\mu k^2 e^4}{2\hbar^2 n^2} = -\frac{m_e k^2 e^4}{4\hbar^2 n^2}}$